American Journal of Engineering Research (AJER)
E-ISSN: 2320-0847 p-ISSN: 2320-0936
Volume-8, Issue-1, pp-10-14
www.ajer.org
Research Paper Open Access

An Algorithm for improving convergence rate by decomposition of interval for solving f(x) = 0

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***ABSTRACT**: Solution of Non-linear Equation has always been a problem for researchers and engineers it arises in many fields of engineering. In this paper the main focus is decomposition of the interval which boosts the rate of convergence of bracketing methods. For this purpose, regulafalsi formula is used to decompose to the interval, when the decomposed interval is applied to regulafalsi method it speeds up its convergence to quadratic, few numerical examples are also included in this paper for comparison with existing methods.*

***KEYWORDS:** nonlinear, sub-interval, bracketing methods, convergence, accuracy*

Date of Submission: 21-12-2018 Date of acceptance: 05-01-2019

I. INTRODUCTION

Non-linear equations occur often in problems of engineering and science like decay equation, charlesrichter magnitude of earthquake, van der waal equation , surface-wave formula and many more, their solution has always been a point of interest for researchers and engineers these equations can be solved analytically as well as numerically but their analytic solution is very hard to find and many times even impossible to find so we have to refer to numerical methods although it will give only approximated roots but there is no other way out there are two types of method for solving these equations numerically i.e. closed and open method , closed methods mostly converge but they have slow convergence however open methods coverage faster than closed methods but they can also diverge in many cases ,as far as convergence is concerned it depends on a good initial approximation many researchers have worked in this area by decomposition of interval, [tanakan , 2013] has developed a new algorithm of modified bisection method for solving nonlinear equations in which he initially decomposed the interval by bisection method and further used that in equation of straight line ,[parida ,2006] developed a improved regulafalsi method for enclosing simple zeros of nonlinear equation by creating regula-falsi newton like algorithm , [shaw, 2015] developed an improved regulafalsi method in which he decomposed interval by regulafalsi and used it in proposed algorithm ,[suhadolnik, 2012] proposed a bisection-parabolic method , [sangah , 2016] developed a modified bracketing method in which he used sub-intervals for solving nonlinear equations,[Xinyuan, 2003] created improved regulafalsi method of quadratic convergence [E.Soomro , 2016] developed a multistep derivative free bracketing method by truncating taylor's series , [A.A.Siyal , 2016] developeda hybrid closed algorithm by combining bisection and regulafalsi method [A.A.Siyal , 2017] developed a modified algorithm in which he decomposed interval by finding harmonic mean and used it in regulafalsi method, [umair,2018] developed a second order bracketing method for solving nonlinear equations.

II. METHODOLOGY OF PROPOSED ALGORITHM:

Given an equation f(x) = 0 and an interval [a,b] which holds the condition $f(a) \cdot f(b) < 0$

Step: 01 Find a sub-interval $[X_L, X_U]$

Process for finding X_L and X_U

CASE (i) when $|f(a)|<|f(b)|$

First Find: $d = a - \frac{f(a)*(b-a)}{f(b)-f(a)}$ ---------- (1)

If f (d) < 0 then d=X_L (lower point which lies before the root) thenfindX_U (upper point which lies after the root but closer than point b) by doubling $\frac{f(a)*(b-a)}{f(b)-f(a)}$in d, then:

$X_U = a - 2 * \frac{f(a)*(b-a)}{f(b)-f(a)}$---------- (2)

But If f (d)>0 then d=X_U (upper point which lies after the root) then find X_L (lower point which lies before the root but closer than point a) by halving $\frac{f(a)*(b-a)}{f(b)-f(a)}$ in d, then:

$X_L = a - \frac{f(a)*(b-a)}{2*(f(b)-f(a))}$---------- (3)(root lies within the red line see fig: 1a & 1b)

CASE (ii) If |f (a)|>|f (b)|

First Find: d = b – $\frac{f(b)*(b-a)}{f(b)-f(a)}$---------- (4)

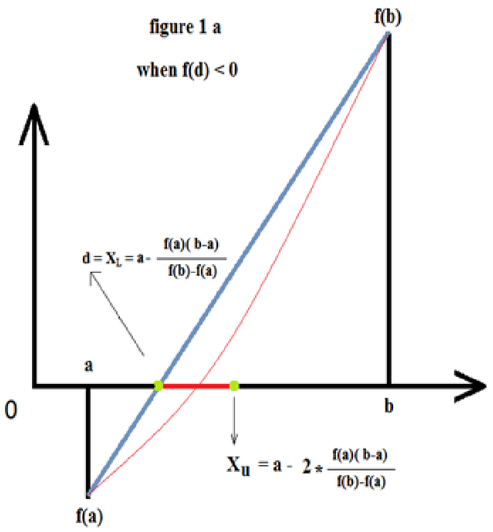
If (d) > 0 then d=X_u (upper point which lies after the root) find X_L (lower point which lies before the root but closer than a)by doubling $\frac{f(b)*(b-a)}{f(b)-f(a)}$in d, then

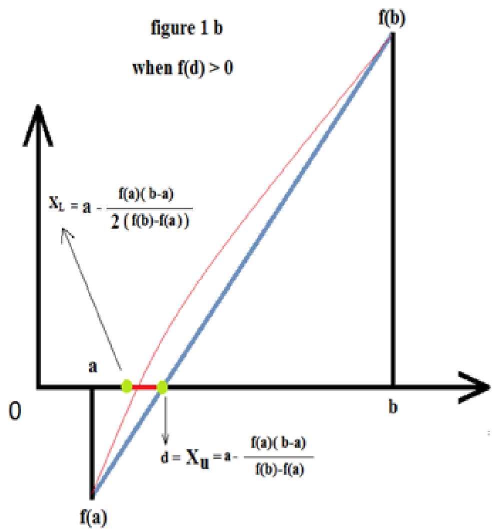
$X_L = b - \frac{2*f(b)*(b-a)}{f(b)-f(a)}$ ---------- (5)

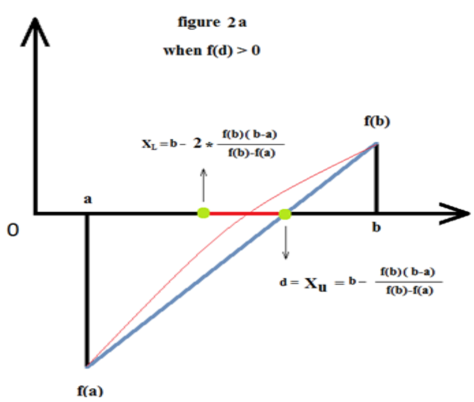
But If f (d) < 0 then d=X_L (lower point which lies before the root) we will find X_U (upperpoint which lies after the root but closer than point b) by halving $\frac{f(b)*(b-a)}{f(b)-f(a)}$ in d,

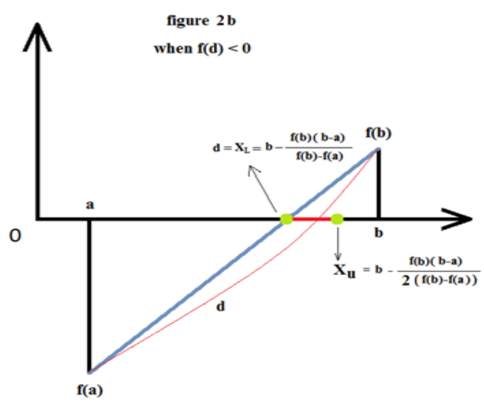
$X_U = b - \frac{f(b)*(b-a)}{2(f(b)-f(a))}$---------- (6)(root lies within the red line see fig: 2a & 2b)

Step: 02 Use sub-interval in existing bracketing methods









III. ORDER OF CONVERGENCE FOR PROPOSED METHOD

When the decomposed interval is applied to regulafalsi method it boosts up it order from linear to quadratic.
Let x_n, x_{n+1}, x_{n+2} be the approximations, α be the root and h_n, h_{n+1}, h_{n+2}be the errors then,

$x_n - \alpha = h_n \Rightarrow x_n = \alpha + h_n$
$x_{n+1} - \alpha = h_{n+1} \Rightarrow x_{n+1} = \alpha + h_{n+1}$
$x_{n+2} - \alpha = h_{n+2} \Rightarrow x_{n+2} = \alpha + h_{n+2}$
$X_{n+2} = \frac{X_l f(X_u) - X_u f(X_l)}{f(X_u) - f(X_l)}$ (7)

Case: 1a

$X_l = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)} = \frac{(\alpha+h_n)f(\alpha+h_{n+1}) - (\alpha+h_{n+1})f(\alpha+h_n)}{f(\alpha+h_{n+1}) - f(\alpha+h_n)}$

$X_l = \alpha + h_n h_{n+1} C = \alpha + e_n$ (8)

Where $e_n = h_n h_{n+1} C$

$$X_u = x_n - \frac{2 * f(x_n)(x_{n+1} - x_n)}{f(x_{n+1}) - f(x_n)} = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)} - \frac{f(x_n)(x_{n+1} - x_n)}{f(x_{n+1}) - f(x_n)}$$

$$X_u = \alpha + \frac{h_n f(\alpha + h_{n+1}) - h_{n+1} f(\alpha + h_n)}{f(\alpha + h_{n+1}) - f(\alpha + h_n)} - \frac{f(\alpha + h_n)(\alpha + h_{n+1} - (\alpha + h_n))}{f(h_{n+1}) - f(h_n)}$$

$X_u = \alpha + h_n h_{n+1} C - h_n = \alpha + e_{n+1}$ (9)

Where $e_{n+1} = h_n h_{n+1} C - h_n$

By putting values of eq (8) and eq (9) ineq (7) we get

$$X_{n+2} = \frac{(\alpha + e_n) f(\alpha + e_{n+1}) - (\alpha + e_{n+1}) f(\alpha + e_n)}{f(\alpha + e_{n+1}) - f(\alpha + e_n)}$$

$$\alpha + h_{n+2} = \alpha + \frac{e_n f(\alpha + e_{n+1}) - e_{n+1} f(\alpha + e_n)}{f(\alpha + e_{n+1}) - f(\alpha + e_n)}$$

$$h_{n+2} = e_n e_{n+1} C$$

$$h_{n+2} = (h_n h_{n+1} C)(h_n h_{n+1} C - h_n) C$$

$$h_{n+2} = h_n^2 h_{n+1} C \quad (10)$$

Case: 1b

$$X_l = x_n - \frac{f(x_n)(x_{n+1} - x_n)}{2(f(x_{n+1}) - f(x_n))} = \frac{(x_n) f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)} + \frac{f(x_n)(x_{n+1} - x_n)}{2(f(x_{n+1}) - f(x_n))}$$

$$X_l = \alpha + \frac{h_n f(\alpha + h_{n+1}) - h_{n+1} f(\alpha + h_n)}{f(\alpha + h_{n+1}) - f(\alpha + h_n)} + \frac{f(h_n)(h_{n+1} - h_n)}{2(f(h_{n+1}) - f(h_n))}$$

$X_l = \alpha + h_n h_{n+1} C + \frac{h_n}{2} = \alpha + e_{n+2}$ (11)

Where $e_{n+2} = h_n h_{n+1} C + \frac{h_n}{2}$

$X_u = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)} = \alpha + h_n h_{n+1} C = \alpha + e_n$ (12)

Where $e_n = h_n h_{n+1} C$

By putting values of eq (11) and eq (12) in eq (7) we get

$$h_{n+2} = e_{n+2} e_n C$$

$$h_{n+2} = (h_n h_{n+1} C + \frac{h_n}{2})(h_n h_{n+1} C) C$$

$$h_{n+2} = h_n^2 h_{n+1} C \quad (13)$$

Let $h_{n+1} = A h_n^k$

$h_n = A^{-\frac{1}{k}} h_{n+1}^{\frac{1}{k}}$

$h_{n+2} = A h_{n+1}^k$

Put values of h_n, h_{n+1} and h_{n+2} in eq(10) or (13)

$$A h_{n+1}^k = (A^{-\frac{1}{k}} h_{n+1}^{\frac{1}{k}})^2 h_{n+1}$$

$$k = \frac{2}{k} + 1$$

$$k = 2$$

Case: 2a

$$X_l = x_{n+1} - 2 * \frac{f(x_{n+1})(x_{n+1} - x_n)}{(f(x_{n+1}) - f(x_n))} = \frac{(x_n) f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)} - \frac{f(x_{n+1})(x_{n+1} - x_n)}{(f(x_{n+1}) - f(x_n))}$$

$$X_l = \alpha + \frac{h_n f(\alpha + h_{n+1}) - h_{n+1} f(\alpha + h_n)}{f(\alpha + h_{n+1}) - f(\alpha + h_n)} - \frac{f(h_{n+1})(h_{n+1} - h_n)}{(f(h_{n+1}) - f(h_n))}$$

$X_l = \alpha + h_n h_{n+1} C - h_{n+1} = \alpha + e_{n+3}$ (14)

Where $e_{n+3} = h_n h_{n+1} C - h_{n+1}$

$X_u = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)}$

$X_u = \alpha + h_n h_{n+1} C = \alpha + e_n$ (15)

Where $e_n = h_n h_{n+1} C$

By putting value of eq (14) and eq (15) in eq (7) we get

$$h_{n+2} = e_{n+3} e_n C$$

$$h_{n+2} = (h_n h_{n+1} C - h_{n+1})(h_n h_{n+1} C)C$$

$h_{n+2} = h_{n+1}^2 h_n C$ (16)

Case: 2b

$X_l = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)}$

$X_l = \alpha + h_n h_{n+1} C = \alpha + e_n$ (17)

Where $e_n = h_n h_{n+1} C$

$$X_u = x_{n+1} - \frac{f(x_{n+1})(x_{n+1} - x_n)}{2 * (f(x_{n+1}) - f(x_n))} = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)} + \frac{f(x_{n+1})(x_{n+1} - x_n)}{2 * (f(x_{n+1}) - f(x_n))}$$

$X_u = \alpha + h_n h_{n+1} C + \frac{h_{n+1}}{2} = \alpha + e_{n+4}$ (18)

Where $e_{n+4} = h_n h_{n+1} C + \frac{h_{n+1}}{2}$

By putting value of eq (17) and eq (18) in eq (7) we get

$h_{n+2} = h_n h_{n+1}^2 C$ (19)

Let $h_{n+1} = A h_n^k$

$h_n = A^{-\frac{1}{k}} h_{n+1}^{\frac{1}{k}}$

$h_{n+2} = A h_{n+1}^k$

Put values of h_n, h_{n+1} and h_{n+2} in eq (19)

$$A h_{n+1}^k = (A^{-\frac{1}{k}} h_{n+1}^{\frac{1}{k}})(h_{n+1})^2$$

$$k = \frac{1}{k} + 2$$

$$k^2 - 2k - 1 = 0$$

$$k = 1 + \sqrt{2}$$

Hence the proposed method has at least quadratic convergence

IV. RESULTS AND CONCLUSIONS

To justify the proposed algorithm few examples are tested using bisection method and regulafalsi method and compared with bisection and regulafalsi through sub-interval, it has been observed that sub-interval improves both methods in terms of accuracy, error and also reduces no of iterations.

Function	Interval	Method	Iterations	x	|f(x)|
$2x - \log(x) - 7 = 0$	[1,10]	B.M	54	4.219906483780380	1.77e-15
		R.F.M	12	4.219906483780380	1.77e-15
		B.M*	04	4.219906483780381	8.88e-16
		R.F.M*	03	4.219906483780381	8.88e-16
$e^x - 2x - 2 = 0$	[0,2]	B.M	51	1.67834699001666	1.77e-15
		R.F.M	25	1.678346990016660	1.33e-15
		B.M*	06	1.678346990016661	8.88e-16
		R.F.M*	06	1.678346990016661	8.88e-16
$x^2 - \sin^2 x - 1 = 0$	[-1,2]	B.M	52	1.40449164821534	2.89e-15
		R.F.M	33	1.404491648215340	1.89e-15
		B.M*	09	1.404491648215341	3.33e-16
		R.F.M*	08	1.404491648215341	3.33e-16
$x^4 - x^3 - x^2 + x + 1 = 0$	[1,2]	B.M	50	1.512876396864093	9.32e-15
		R.F.M	53	1.512876396864094	2.88e-16
		B.M*	06	1.512876396864095	6.66e-16
		R.F.M*	06	1.512876396864095	6.66e-16
$x\sin x - \cos x = 0$	[0,1]	B.M	50	0.86033358901938	4.45e-16
		R.F.M	12	0.860333589019379	1.11e-16
		B.M*	04	0.860333589019379	1.11e-16
		R.F.M*	04	0.860333589019379	1.11e-16

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