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Using Lagrange interpolation to determine the milk production amount by the number of milked animals

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ABSTRACT:*In this study, milk production by the number of buffalos produced and milked between the years of 2013-2017 in Turkey's Eskisehir, Ankara and Konya provinces has been examined. Lagrange polynomial was formed to detect milk production by the number of animals milked. With the help of this polynomial, production estimates have also been made for the intermediate values of the independent variable x. It is estimated that in Eskişehir, the average milk amount per 100-140 animals is 107.08-149.88 kg, the average milk amount per 400-750 animals in Ankara is 447.25-837.75 kg, and the average milk amount per 100-260 animals is 128.40-333.84 kg in Konya. The interpolation method has been a good model for estimating production in farming.*

Keywords: *Interpolation, milk, buffalo.*

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I. INTRODUCTION

Milk, which is necessary at every stage of human life, is a good nutrition for macro and micro-nutritional elements. Milk is known for its importance for bone health especially during childhood, pregnancy-lactation, and old age. There are also researches that indicates its relationship with chronic diseases such as obesity, cancer, and hypertension [1-3]. Increasing the consumption of milk and dairy products is recommended by health professionals [2, 4].

According to FAO (Food and Agriculture Organization of the United Nations) statistics of 2016, in world ranking, Turkey ranked 10th with 56 161 tons in the amount of buffalo milk production. India, Pakistan, and China are the top three buffalo milk producers in the world. In terms of buffalo milk yield, Iran comes 1., Greece comes 2., Pakistan comes 3. and Turkey ranking 12th [5].

According to the statistics of TSI (Turkey Statistics Institute) in 2017, the total milk production of 21 223 289 tons is composed of 18 762 319 tons cattle milk, 1 344 779 tons sheep milk, 523 395 tons goat milk and 69 401 tons buffalo milk. 0.327% of this production is buffalo milk [6].

The increased amount of milk production depends on the number of animals being milked. If the number of animals milked is high, an increase in production is expected. Several statistical and mathematical models can be established on how much milk can be produced from a certain number of animals. One of these models is interpolation polynomials.

Functions which cannot be performed analytically and only given numerically with the help of table points can be expressed as analytical statements, when the function is given numerically to solve problems outside the table points. This can be achieved by the function approach and interpolation methods [7].

Interpolation is a mathematical function that estimates the values at locations where no measured values are available [8]. Basic properties of the interpolation polynomial are the existence and the uniqueness of the interpolation polynomials [9].

At the present time, interpolation method is used, whether in agricultural production or defense advanced science and technology research, such as large and medium-sized electromechanical product optimization design, major project design, etc [10].

In this study, it is aimed to estimate the amount of milk by the number of buffalos milked using the interpolation equation determined by the Lagrange interpolation method to estimate the amount of milk corresponding to the desired number of buffalos.

II. MATERIAL

The material of this research consists of the number of milked buffalos and buffalo milk production amount (tons) in the period of 2013-2017 from the website of Turkey Statistics Institution (TSI) in the provinces of Turkey, Eskisehir, Ankara, and Konya. These provinces were chosen since they are more developed in population, economy, industry, agriculture and other aspects compared to other cities in the Central Anatolia Region. Lagrange Interpolation polynomial was obtained and the amount of milk by the number of milked animals was calculated.

In Eskisehir, Ankara and Konya provinces, the number of milked animals and the milk production amount of this period are given in Table 1.

Table 1. Milk production according to the number of milked buffalos (tons)

Years	Eskişehir		Ankara		Konya	
	Number of buffalo milked	Amount of milk production (ton)	Number of buffalo milked	Amount of milk production (ton)	Number of buffalo milked	Amount of milk production (ton)
2013	96	102.848	370	412.843	152	195.168
2014	126	135.227	400	447.247	83	106.572
2015	131	139.988	533	595.674	121	155.364
2016	114	121.894	523	583.878	135	173.34
2017	144	154.273	785	876.8	270	346.68

III. METHOD

Lagrange interpolation is used also when the independent variable intervals are not equal. Assuming x_0, x_1, ..., x_n are different real or complex numbers and y_0, y_1, ..., y_n are corresponding values to these numbers for y=f(x) function.

$p(x_i)=f(x_i)$; i=0, 1, 2, ..., n

a polynomial p(x) is obtained accordingly. This polynomial will be like the following.

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

A system consisting of (n + 1) number of equations with (n + 1) unknowns will arise [11]. Given that at (n+1) different points there are x_0, x_1, ..., x_n. For i=0, 1, 2, ..., n at (n+1) number of points are defined as below.

$$L(x) = \frac{(x - x_0) \dots (x - x_{i-1})(x - x_{i+1}) \dots (x - x_n)}{(x_i - x_0) \dots (x_i - x_{i-1})(x_i - x_{i+1}) \dots (x_i - x_n)}$$

Each of these is a polynomial of an nth degree. These polynomials are called Lagrange polynomials for x_0, x_1, ..., x_n points. The sum of the polynomials of the nth degree is shown below.

$$p(x) = f_0 L_0(x) + f_1 L_1(x) + f_2 L_2(x) + \cdots + f_i L_i(x) + \cdots + f_n L_n(x)$$

Thus, the p (x) polynomial is shown as

$$p(x) = \sum_{i=0}^{n} f_i L_i(x)$$

and this is called Lagrange interpolation [12].

In other words, the Lagrange interpolating polynomial is the polynomial p(x) of degree less than (n-1) that passes through the n points $(x_1, y_1 = f(x_1))$, $(x_2, y_2 = f(x_2))$,...,$(x_n, y_n = f(x_n))$, and is given by

$$f(x) = \textstyle\sum_{j=1}^{n} f_j(x)$$

Where

$$f_j(x) = y_j \prod_{\substack{k=1 \\ k \neq j}}^{n} \frac{x - x_k}{x_j - x_k}$$

written explicitly,

$$f(x) = \frac{(x - x_2)(x - x_3) \dots (x - x_n)}{(x_1 - x_2)(x_1 - x_3) \dots (x_1 - x_n)} y_1$$

$$+ \frac{(x - x_1)(x - x_3) \dots (x - x_n)}{(x_2 - x_1)(x_2 - x_3) \dots (x_2 - x_n)} y_2$$

⋮

$$+\frac{(x-x_1)(x-x_2)\ldots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})}y_n$$

The formula was first published by Waring (1779), rediscovered by Euler in 1783, and published by Lagrange in 1795 [13].

IV. RESULTS

Milk production amount equations were created separately according to the number of buffalos milked in the provinces of Eskisehir, Ankara, and Konya in Turkey. These equations were obtained by using Lagrange interpolation.

Eskişehir

The Lagrange polynomial for the amount of milk produced in the province of Eskisehir according to the number of milked buffalos is as follows.

$$\begin{aligned} f(x) &= \frac{(x-126)(x-131)(x-114)(x-144)}{(96-126)(96-131)(96-114)(96-144)}102.848 \\ &+ \frac{(x-96)(x-131)(x-114)(x-144)}{(126-96)(126-131)(126-114)(126-144)}135.227 \\ &+ \frac{(x-96)(x-126)(x-114)(x-144)}{(131-96)(131-126)(131-114)(131-144)}139.988 \\ &+ \frac{(x-96)(x-126)(x-131)(x-144)}{(114-96)(114-126)(114-131)(114-144)}121.894 \\ &+ \frac{(x-96)(x-126)(x-131)(x-114)}{(144-96)(144-126)(144-131)(144-114)}154.273 \end{aligned}$$

Lagrange polynomial is obtained at 4th degree due to the number of observations of x and y variables being 5. When the necessary calculations and adjustments were made, the following equation was obtained. The Lagrange interpolation which consists of the coefficients of the function determined as x is as follows.

$$f(x) = 1.8762e-05\ x^4 - 0.00908\,x^3 + 1.6361\,x^2 - 128.9928\ x + 3847.6084$$

Lagrange interpolation program written in the MATLAB program is presented in Table 2.

Table 2. MATLAB program for milk amount according to the number of buffalo milked in Eskisehir

```
x=[96 126 131 114 144];
y=[102.848 135.227 139.988 121.894 154.273];
x(1);x(2);x(3);x(4);x(5);
y(1);y(2);y(3);y(4);y(5);
k1=[1 -x(1)];k2=[1 -x(2)];k3=[1 -x(3)];k4=[1 -x(4)];k5=[1 -x(5)];
m1=conv(conv([1 -x(2)],[1 -x(3)]),conv([1 -x(4)],[1 -x(5)]))
payda1=(x(1)-x(2))*(x(1)-x(3))*(x(1)-x(4))*(x(1)-x(5));
fonk1=m1*y(1)/payda1
m2=conv(conv([1 -x(1)],[1 -x(3)]),conv([1 -x(4)],[1 -x(5)]))
payda2=(x(2)-x(1))*(x(2)-x(3))*(x(2)-x(4))*(x(2)-x(5));
fonk2=m2*y(2)/payda2
m3=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(4)],[1 -x(5)]))
payda3=(x(3)-x(1))*(x(3)-x(2))*(x(3)-x(4))*(x(3)-x(5));
fonk3=m3*y(3)/payda3
m4=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(3)],[1 -x(5)]))
payda4=(x(4)-x(1))*(x(4)-x(2))*(x(4)-x(3))*(x(4)-x(5));
fonk4=m4*y(4)/payda4
m5=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(3)],[1 -x(4)]))
payda5=(x(5)-x(1))*(x(5)-x(2))*(x(5)-x(3))*(x(5)-x(4));
fonk5=m5*y(5)/payda5
t=fonk1+fonk2+fonk3+fonk4+fonk5
xg=100:5:140;
yg=interp1(x,y,xg,'lagrange')
plot(xg,yg,'*')
```

```
format long
poly2str(t,'x')
format long
```

According to this polynomial, the estimate of milk production from 100 to 140 is presented in Table 3.

Table 3. Estimated amount of buffalo milk production for Eskisehir

Number of milked buffalo	Milk production (Ton)
100	107.08
105	112.37
110	117.66
115	123.01
120	128.56
125	134.12
130	139.04
135	144.38
140	149.88

According to the results obtained in Table 3, the average annual milk amount per buffalo was 1070.46 kg in Eskisehir. The graph for the amount of milk estimated by the number of animals milked is shown in Figure 1.

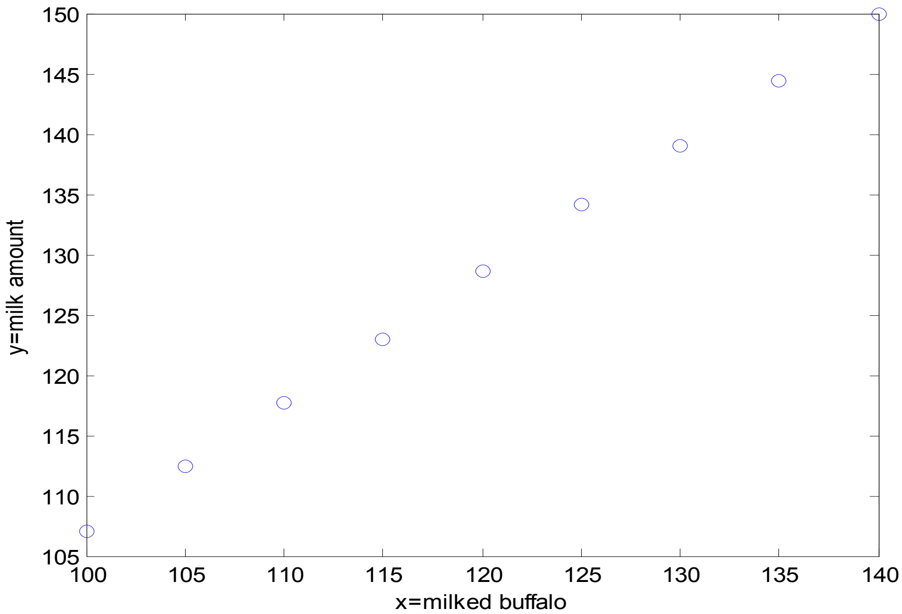

Figure 1. Milk production by the number of animals milked (tons)

Ankara

The Lagrange polynomial regarding the amount of milk produced according to the number of buffalos milked in Ankara province is as follows.

$$
\begin{aligned}
f(x) = {} & \frac{(x-400)(x-533)(x-523)(x-785)}{(370-400)(370-533)(370-523)(370-785)} 412.843 \\
& + \frac{(x-370)(x-533)(x-523)(x-785)}{(400-370)(400-533)(400-523)(400-785)} 447.247 \\
& + \frac{(x-370)(x-400)(x-523)(x-785)}{(533-370)(533-400)(533-523)(533-785)} 595.674 \\
& + \frac{(x-370)(x-400)(x-533)(x-785)}{(523-370)(523-400)(523-533)(370-785)} 583.878 \\
& + \frac{(x-370)(x-400)(x-533)(x-523)}{(785-370)(785-400)(785-533)(785-523)} 876.8
\end{aligned}
$$

A Lagrange polynomial of 4th degree was obtained. When the necessary calculations and adjustments were made, the following equation was obtained.
The Lagrange interpolation which consists of the coefficients of the function determined as x is as follows.

$$f(x) = -1.5887e-08\ x^4 + 3.3625e-05\,x^3 - 0.0259\,x^2 + 9.7626\ x - 1058.9627$$

Lagrange interpolation program written in the MATLAB program is shown in Table 4.

Table 4. MATLAB program for milk amount according to the number of buffalo milked in Ankara

```
x=[370 400 533 523 785];
y=[412.843 447.247 595.674 583.878 876.8];
x(1);x(2);x(3);x(4);x(5);
y(1);y(2);y(3);y(4);y(5);
k1=[1 -x(1)];k2=[1 -x(2)];k3=[1 -x(3)];k4=[1 -x(4)];k5=[1 -x(5)];
m1=conv(conv([1 -x(2)],[1 -x(3)]),conv([1 -x(4)],[1 -x(5)]))
payda1=(x(1)-x(2))*(x(1)-x(3))*(x(1)-x(4))*(x(1)-x(5));
fonk1=m1*y(1)/payda1
m2=conv(conv([1 -x(1)],[1 -x(3)]),conv([1 -x(4)],[1 -x(5)]))
payda2=(x(2)-x(1))*(x(2)-x(3))*(x(2)-x(4))*(x(2)-x(5));
fonk2=m2*y(2)/payda2
m3=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(4)],[1 -x(5)]))
payda3=(x(3)-x(1))*(x(3)-x(2))*(x(3)-x(4))*(x(3)-x(5));
fonk3=m3*y(3)/payda3
m4=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(3)],[1 -x(5)]))
payda4=(x(4)-x(1))*(x(4)-x(2))*(x(4)-x(3))*(x(4)-x(5));
fonk4=m4*y(4)/payda4
m5=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(3)],[1 -x(4)]))
payda5=(x(5)-x(1))*(x(5)-x(2))*(x(5)-x(3))*(x(5)-x(4));
fonk5=m5*y(5)/payda5
t=fonk1+fonk2+fonk3+fonk4+fonk5
xg=400:10:750;
yg=interp1(x,y,xg,'lagrange')
plot(xg,yg,'o')
format long
poly2str(t,'x')
format long
```

According to this polynomial, the estimate of milk production from 400 to 750 is given in Table 5.

Table 5. Estimated amount of buffalo milk production for Ankara

Number of milked buffalo	Milk amount (tons)
400	447.25
450	502.79
500	558.33
550	614.64
600	670.42
650	726.20
700	781.98
750	837.75

According to the results obtained in Table 5, the average annual milk amount per buffalo was 1117.29 kg in Ankara.
The graph for the amount of milk estimated by the number of animals milked in Ankara province is shown in Figure 2.

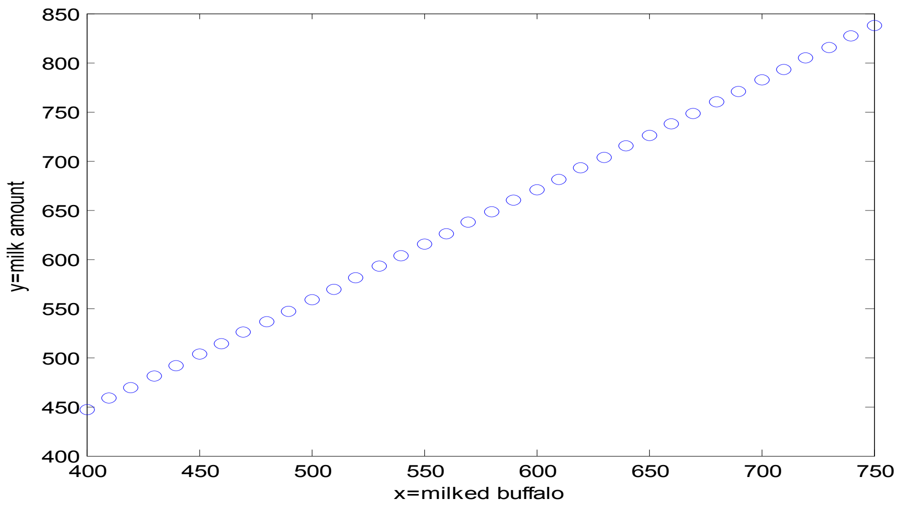

Figure 2. Milk production by the number of milked buffalos in Ankara (tons)

Konya

The Lagrange polynomial regarding the amount of milk produced according to the number of buffalos milked in Konya province is as follows.

$$f(x) = \frac{(x-83)(x-121)(x-135)(x-270)}{(152-83)(152-121)(152-135)(152-270)}195.168 + \frac{(x-152)(x-121)(x-135)(x-270)}{(83-152)(83-121)(83-135)(83-270)}106.572 + \frac{(x-152)(x-83)(x-135)(x-270)}{(121-152)(121-83)(121-135)(121-270)}155.364 + \frac{(x-152)(x-83)(x-121)(x-270)}{(135-152)(135-83)(135-121)(135-270)}173.34 + \frac{(x-152)(x-83)(x-121)(x-135)}{(270-152)(270-83)(270-121)(270-135)}346.68$$

A Lagrange polynomial of 4th degree was obtained here too. When the polynomial is adjusted, the following equation is obtained.

The Lagrange interpolation which consists of the coefficients of the function determined as x is as follows.

$$f(x) = -6.3527e-21\ x^4 - 1.2468e-18\,x^3 + 1.0408e-15\,x^2 + 1.284\ x + 9.9476e-13$$

Lagrange interpolation program written in the MATLAB program is given in Table 6.

Table 6. MATLAB program for milk amount according to the number of buffalo milked in Konya

```
x=[152 83 121 135 270];
y=[195.168 106.572 155.364 173.34 346.68];
x(1);x(2);x(3);x(4);x(5);
y(1);y(2);y(3);y(4);y(5);
k1=[1 -x(1)];k2=[1 -x(2)];k3=[1 -x(3)];k4=[1 -x(4)];k5=[1 -x(5)];
m1=conv(conv([1 -x(2)],[1 -x(3)]),conv([1 -x(4)],[1 -x(5)]))
payda1=(x(1)-x(2))*(x(1)-x(3))*(x(1)-x(4))*(x(1)-x(5));
fonk1=m1*y(1)/payda1
m2=conv(conv([1 -x(1)],[1 -x(3)]),conv([1 -x(4)],[1 -x(5)]))
payda2=(x(2)-x(1))*(x(2)-x(3))*(x(2)-x(4))*(x(2)-x(5));
fonk2=m2*y(2)/payda2
m3=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(4)],[1 -x(5)]))
payda3=(x(3)-x(1))*(x(3)-x(2))*(x(3)-x(4))*(x(3)-x(5));
fonk3=m3*y(3)/payda3
```

```
m4=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(3)],[1 -x(5)]))
payda4=(x(4)-x(1))*(x(4)-x(2))*(x(4)-x(3))*(x(4)-x(5));
fonk4=m4*y(4)/payda4
m5=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(3)],[1 -x(4)]))
payda5=(x(5)-x(1))*(x(5)-x(2))*(x(5)-x(3))*(x(5)-x(4));
fonk5=m5*y(5)/payda5
t=fonk1+fonk2+fonk3+fonk4+fonk5
xg=100:20:260;
yg=interp1(x,y,xg,'lagrange')
plot(xg,yg,'o')
format long
poly2str(t,'x')
format long
```

According to this polynomial, the estimate of milk production from 100 to 260 is given in Table 7.

Table 7. Estimated amount of buffalo milk production for Konya

Number of milked buffalo	Milk amount (tons)
100	128.40
120	154.80
140	179.76
160	205.44
180	231.12
200	256.80
220	282.48
240	308.16
260	333.84

According to the results obtained in Table 5, the average annual milk amount per buffalo was 1284.67 kg in Konya. The graph for the amount of milk estimated by the number of animals milked in Konya province is shown in Figure 3.

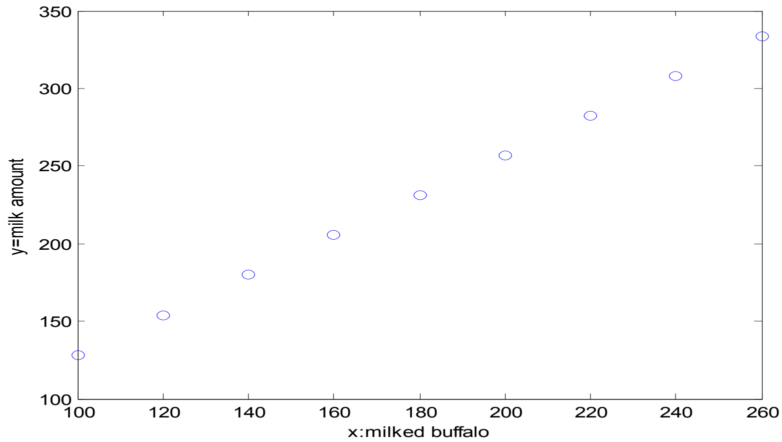

Figure 3. Milk production by the number of milked buffalos in Konya (tons)

V. CONCLUSION

In this study, the yield of milk produced by per milked buffalo in the provinces of Eskisehir, Ankara, and Konya in Turkey was determined as in order of 1070.46, 1117.29 and 1284.67 kg. As a result of the Lagrange polynomial obtained at the 4th degree, the estimated values and real values of milk production amount is very close to each other. This shows that the Lagrange polynomial is a very good interpolation method.

REFERENCES

[1]. Black, R. E., Williams, S. M., Jones, I. E, Goulding, A. 2002. Children who avoid drinking cow milk have low dietary calcium intakes and poor bone health. American Journal of Clinical Nutrition. 76: 675-80.

[2]. Heaney, P., McCarron, D., Dawson-Huges, B., et al. 1999. Dietary changes in favourably affect bone remodeling in older adults. Journal of the American Dietetic Association. 99: p. 1128-1133.

[3]. Miller, G. D., Jarvis, K. J., McBean, L. D. 2000. Handbook of Dairy Foods and Nutrition. In: Jensen R. G., Kroger M., editors. The importance of milk and milk products in the diet. CRC Press, New York, pp. 4-24.
[4]. Weinberg L. G., Louıse A., Berner, Grones J. E. 2004. Utrient Contributions of Dairy Foods in the United States, Continuing Survey of Food Intakes by Individuals 1994-1996, 1998. J Am Diet Assoc., 104: 895-902.
[5]. FAO (Food and Agriculture Organization of the United Nations), 2016. Livestock Processed. Milk, whole fresh buffalo. http://www.fao.org/faostat/en/#data/QL (Accessed to 16.06.2018).
[6]. TSI (Turkey Statistics Institute), 2017. Number of animals milked by types and races and quantity of milk production. http://www.tuik.gov.tr/PreTablo.do?alt_id=1002 (Accessed to: 11.06.2018).
[7]. Akın, Ö. (1998). Nümerik Analiz, Ankara Üniversitesi Fen Fakültesi Ders Kitapları Yayın No: 149, Ankara, s. 123.
[8]. Baillargeon, S. 2005. Kriging review of the theory and application to the interpolation of precipitation data. Thesis University of Laval (Quebec), 137 p.
[9]. Mingyou, H., Bo, L., Tao, X. 2005. Numerical Calculation Method [M], Science publishing company.
[10]. Qingyang, L., Nengchao, W., Dayi, Yi. 2008. Numerical Analysis.5th Ed [M], QingHua University publishing company.
[11]. Türker, E. S., Can, E. 1997. Bilgisayar Uygulamalı Sayısal Analiz Yöntemleri. Değişim Yayınları, Adapazarı, s. 234-239.
[12]. Prasad, D. 2006. An Introduction to Numerical Analysis, Norasa Publishing House, New Delhi.
[13]. Jeffreys, H., Jeffreys, B. S. 1988. Lagrange's Interpolation Formula. §9.011 in Methods of Mathematical Physics, 3rd ed. Cambridge, England: Cambridge University Press, p260.

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