

The Relation Between Incident And Medium Electromagnetic Field And Left Handed Materials

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ABSTRACT: The electric and magnetic properties of matter play an important role in the optical properties of matter. Thus one expects any model that describes left handed materials to rely heavily on the notion of electromagnetic field. In this work the notion of medium electric and magnetic fields is used to construct a model that describes the nature and behavior of left handed materials. The model shows that the left handed material atoms should be in the form of electric and magnetic dipoles having certain values and certain orientations with respect to the internal field. The model expresses the refractive index in terms of the electric and magnetic permeability components parallel and perpendicular to the external electric and magnetic fields.

Keywords: refractive index left handed material, electric permittivity, magnetic permeability.

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I. INTRODUCTION

In the March of 2000, scientists at the University of California at San Diego announced their production of the first "Left-Handed Material." These materials are exciting because of their unusual optical properties; in a left-handed material, light seems to propagate opposite the direction of energy flow [1,2]. This leads to a negative index of refraction and reversed Doppler shift for radiation! The materials were called "left-handed", because the electric and magnetic fields propagate in a direction given by a left-hand rule: put the thumb of your left hand in the direction of the electric field, the fingers of your left hand in the direction of the magnetic field, and your left palm will point in the direction of motion [3,4]. (Conventional materials follow a right-hand rule, as described in the module - same assignments of thumb, fingers, and palm but using the right hand). But "left-handed" is used to describe a different set of properties when applied to particles (see, for example, from), so some prefer to identify these materials as "negative index" materials [5,6]. Snell's law has long been used to calculate angles and refractive indices for refractive materials. What we didn't know until recently, however, is that these indices of refraction can be negative! As light enters a left-handed material from a right-handed material (a material we are already familiar with like glass or air) the light will refract, but the refracted ray in a left-handed material will be a mirror image of the refracted ray we typically see in right-handed materials. Refraction is not the only effect where left-hand materials give an unexpected result. Light from a source traveling toward a detector through normal (right-handed) materials appears "blue-shifted" because of the Doppler Effect. As the waves approach an object, they bunch up, decreasing the wavelength as measured by the object. But in a left-handed material, light from a source traveling toward an object is red-shifted, appearing to have a longer wavelength! For more information about the Doppler Effect, refer to from materials [7]. To this date, no one actually knows why the materials act the way they do. All that is known is that the material has a negative electric permittivity and negative magnetic permeability, which could explain why the Doppler shift and Snell effect are inverted [8]. Unfortunately, no one has yet explained why the material's permeability and permittivity would be negative. The links below provide more information about this exciting new class of materials. You might want to bookmark footnote, that page will be updated as more developments areas. This work is devoted to construct a model that shows the atomic nature of left handed material this is done in section2.Sections 3 and 4 are devoted for discussion and conclusion.

II. THEORETICAL MODEL

The complex refractive index can be written as

$$n_1 = c^2 \left(\frac{\mu_1 \varepsilon_2 + \mu_2 \varepsilon_1}{2n_2} \right) \quad (1)$$

The left handed materials have negative refractive index, i.e.

$$n_1 = - \quad (2)$$

To see how this condition is satisfied, it is better to write the total magnetic field intensity H and flux density in the form

$$\underline{B} = \mu \underline{H} = \mu_0 (H_e + H_m) \quad (3)$$

Where H_e and H_m stands for external and medium field respectively.

The medium field can be spotted into real part parallel to the external field and i.e. imaginary one perpendicular to it ,

$$H_m = (H_{mx} + iH_{my}) \quad (4)$$

The medium field itself can be expressed in terms of external one in the form

$$H_m = c_1 H_e + ic_2 H_e = H_{m1} + iH_{m2} \quad (5)$$

Which can also be rewritten in terms of the angle ϕ

$$H_m = c_0 H_e \cos \phi + ic_0 H_e \sin \phi \quad (6)$$

Thus inserting equation (6) in (3), yields

$$\underline{B} = \mu_0 (1 + c_0 \cos \phi) H_e + i\mu_0 c_0 \sin \phi H_e \quad (7)$$

Therefore

$$\underline{B} = [\mu_0 (1 + c_0 \cos \phi) + i\mu_0 c_0 \sin \phi] H_e \quad (8)$$

But since B can also be written as

$$\underline{B} = \mu H_e = (\mu_1 + i\mu_2) H_e \quad (9)$$

Thus comparing (8) and (9) gives

$$\mu_1 = \mu_0 (1 + c_0 \cos \phi) \quad , \quad \mu_2 = \mu_0 c_0 \sin \phi \quad (10)$$

Similarly the electric flux density \underline{D} and intensity \underline{E} can be written in terms of the external field E_e and medium field E_m .

$$\underline{D} = \varepsilon E_e = \varepsilon_0 (E_e + E_m) \quad (11)$$

$$E_m = (E_{mx} + iE_{my}) \quad (12)$$

$$E_m = c_1 E_e + ic_2 E_e = E_{m1} + iE_{m2} \\ = c_0 E_e \cos \theta + ic_0 E_e \sin \theta \quad (13)$$

$$\underline{D} = \varepsilon_0 (E_e + E_m) \quad (14)$$

$$\underline{D} = [\varepsilon_0 (1 + c_0 \cos \theta) E_e + i\varepsilon_0 c_0 \sin \theta E_e] \quad (15)$$

$$\underline{D} = [\varepsilon_0 (1 + c_0 \cos \theta) + i\varepsilon_0 c_0 \sin \theta] E_e \quad (16)$$

$$\underline{E} = \varepsilon_0 (E_e + c_1 E_e + ic_2 E_e) = \varepsilon_0 [(1 + c_1) + ic_2] E_e \\ = [\varepsilon_1 + i\varepsilon_2] E_e \quad (17)$$

$$\varepsilon_1 = \varepsilon_0 (1 + c_0 \cos \theta) \quad \varepsilon_2 = \varepsilon_0 c_0 \sin \theta \quad (18)$$

In view of equation (1), n_1 is negative if

$$\underline{a.} \quad \mu_1 = - \quad \mu_2 = - \quad \varepsilon_1 = + \quad \varepsilon_2 = + \quad (19)$$

This can satisfied, according to equation (10) ,when

$$\mu_1 = \mu_0 (1 + c_0 \cos \phi) < 0$$

$$\mu_2 = \mu_0 c_0 \sin \phi < 0$$

For : $\mu_1 = -$ this requires $(1 + c_0 \cos \phi) < 0$

Thus

$$|c_0 \cos \phi| > 1 \quad c_0 \cos \phi = - \quad \phi = - \quad (20)$$

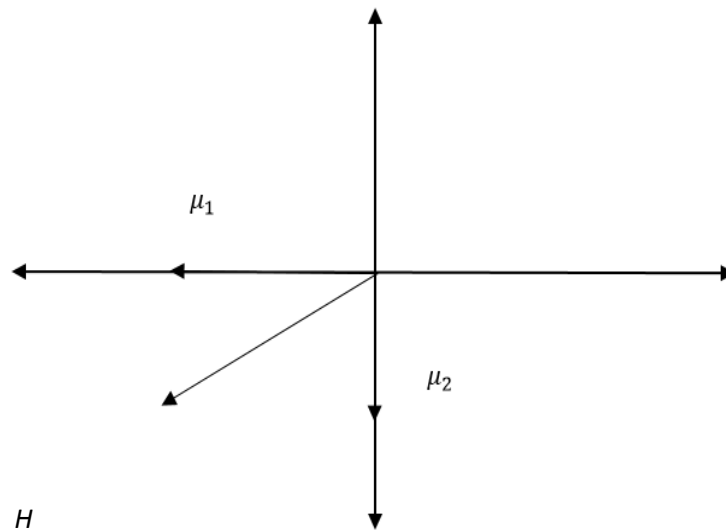
For: $\mu_2 = -$ this requires

$$\mu_2 = \mu_0 c_0 \sin \phi \\ \mu_2 = - \quad \text{if} \quad c_0 \sin \phi < 0$$

$$|c_0 \sin \phi| > 1$$

Thus

$$c_0 \sin \phi = - \quad \sin \phi = - \quad \phi = - \quad (21)$$



In view of equation (1), n_1 is negative if

b. $\mu_1 = -$ $\epsilon_2 = +$ $\mu_2 = +$ $\epsilon_1 = -$
 This can be satisfied, according to equation (10), when

$$\mu_1 = \mu_0(1 + c_0 \cos \phi) < 0$$

$$\mu_2 = \mu_0 c_0 \sin \phi < 0$$

For: $\mu_1 = -$ This requires that $(1 + c_0 \cos \phi) < 0$

Thus

$$|c_0 \cos \phi| > 1 \quad c_0 \cos \phi = - \quad \phi = -$$

b. According to equation (1), n_1 is negative also if

$$\epsilon_1 = - \quad \epsilon_2 = - \quad (22)$$

According to equation (18) this requires

$$\epsilon_0 (1 + c_0 \cos \theta) < 0$$

Thus

$$\cos \theta < -\frac{1}{c_0}$$

or

$$\cos \theta = - \quad (23)$$

$$c_0 < 0$$

It also requires

$$\epsilon_0 c_0 \sin \theta = -$$

This requires

$$\sin \theta = - \quad (24)$$

This means that n_1 is negative when

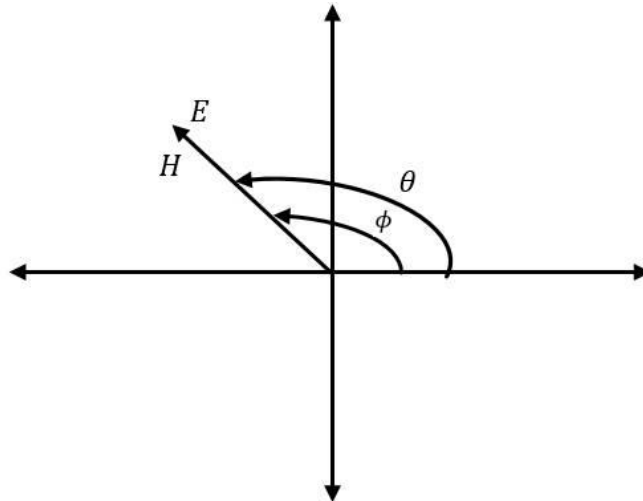
$$180^\circ < \theta < 270^\circ \quad (25)$$

c. Equation (1) indicates that n_1 is negative if

$$\mu_2 = + \quad \epsilon_2 = + \quad \mu_1 = - \quad \epsilon_1 = - \quad (26)$$

This requires

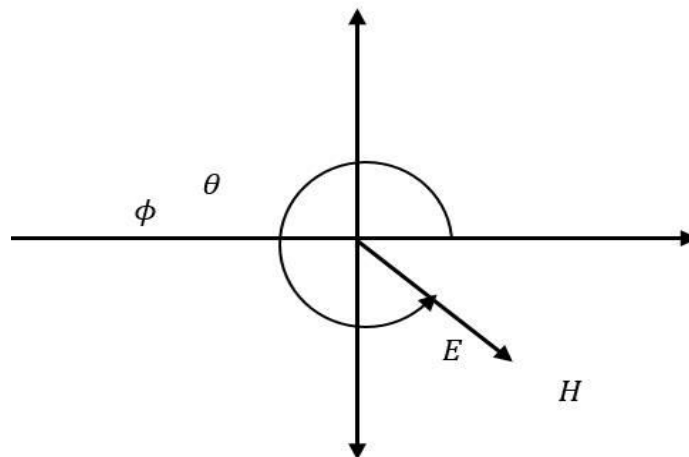
$$90^\circ < \phi < 180^\circ \quad 90^\circ < \theta < 180^\circ \quad (27)$$



d. Also n_1 is negative ,according to equation (1) when

$$\begin{matrix} \epsilon_2 = - & \mu_2 = - \\ \epsilon_1 = + & \mu_1 = + \end{matrix} \quad (28)$$

$$270^\circ < \theta < 360^\circ \quad (29)$$



The refractive index n_1 given according to equation (1) by

$$n_1 = c^2 \left(\frac{\mu_1 \epsilon_2 + \mu_2 \epsilon_1}{2n_2} \right)$$

for n_2 positive n_1 can be negative if

$$|\mu_2 \epsilon_1| > |\mu_1 \epsilon_2| \quad (30)$$

This can be satisfied when

$$\mu_1 = + \quad \epsilon_2 = + \quad \mu_2 = + \quad \epsilon_1 = - \quad (31)$$

$$\mu_2 = - \quad \epsilon_1 = + \quad \mu_1 = + \quad \epsilon_2 = + \quad (32)$$

Also when

$$|\mu_1 \epsilon_2| > |\mu_2 \epsilon_1| \quad (33)$$

n_1 is negative when

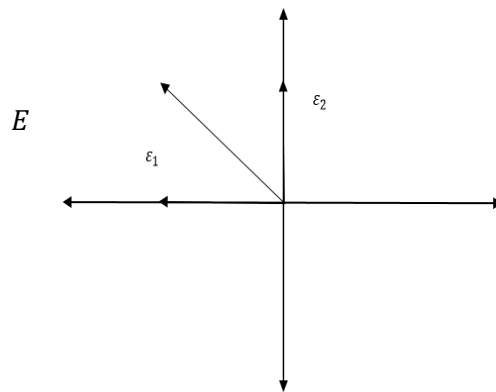
$$\mu_1 = - \quad \epsilon_2 = + \quad \mu_2 = + \quad \epsilon_1 = + \quad (34)$$

$$\mu_1 = + \quad \epsilon_2 = - \quad \mu_2 = + \quad \epsilon_1 = + \quad (35)$$

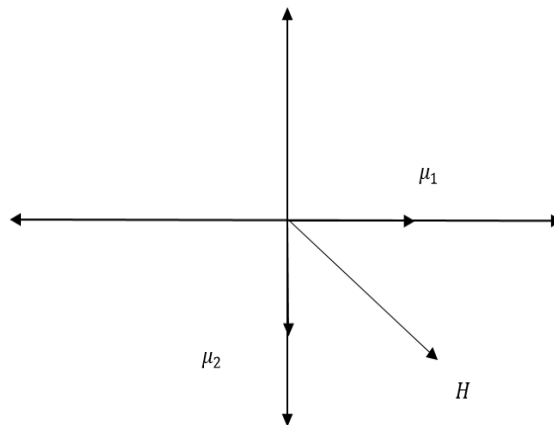
Condition (31) requires that

$$\text{For: } \epsilon_1 = - \quad (1 + c_0 \cos \theta) < 0 \quad |c_0 \cos \theta| > 1$$

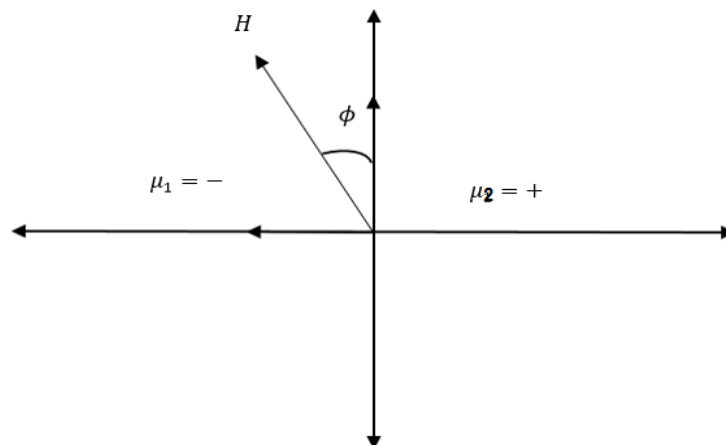
$$c_0 \cos \theta = - \quad \cos \theta = - \quad \theta = -$$



Condition (32) requires that
 For: $\mu_2 = -$ this requires $\mu_2 = \mu_0 c_0 \sin \phi$
 $\mu_2 = -$ if $c_0 \sin \phi < 0$
 Thus
 $|c_0 \sin \phi| > 1$
 $c_0 \sin \phi = -$ $\sin \phi = -$
 Which mean that $\phi = -$

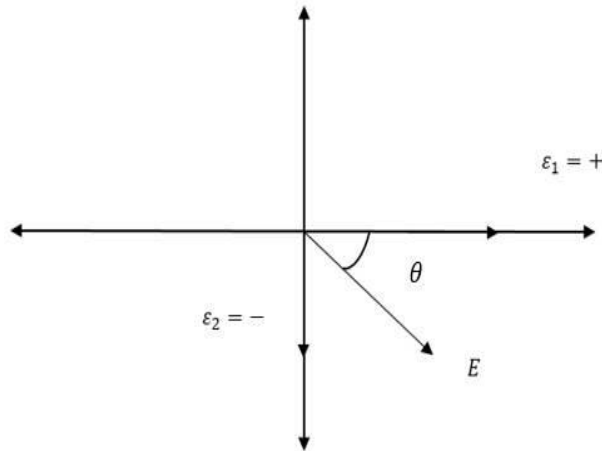


Condition (34) requires that
 For : $\mu_1 = -$ this requires $(1 + c_0 \cos \phi) < 0$ $|c_0 \cos \phi| > 1$
 $c_0 \cos \phi = -$ $\cos \phi = -$ $\phi = +$



Condition (35) requires that

For : $\epsilon_2 = -$ this requires $(1 + c_0 \sin \theta) < 0$ $|c_0 \sin \theta| > 1$
 Thus $c_0 \sin \theta = -$ $\sin \theta = -$ $\theta = -$



III. DISCUSSION

The condition to have left handed materials are studied here on the basis of the angles θ and ϕ subtended by the medium generated electric and magnetic fields with respect to (w.r.t) the external incident electric and magnetic fields respectively. According to equations (20) and (21) the material become left handed when μ_1, μ_2 , are negative. This requires the angle ϕ subtended by the medium magnetic field with respect to the external one to be in the range

$$180^\circ < \phi < 270^\circ$$

Equation (22) and (25) shows that left handed materials requires the angle subtended by the medium electric field with respect to electric field is given by

$$180^\circ < \theta < 270^\circ$$

The materials can also be left handed, according to equation (26) if μ_1 and ϵ_1 are negative this requires [see equation (27)]

$$90^\circ < \theta < 180^\circ$$

$$90^\circ < \phi < 180^\circ$$

According to equations (28) and (29) the material is left handed when

$$270^\circ < \theta < 360^\circ$$

$$270^\circ < \phi < 360^\circ$$

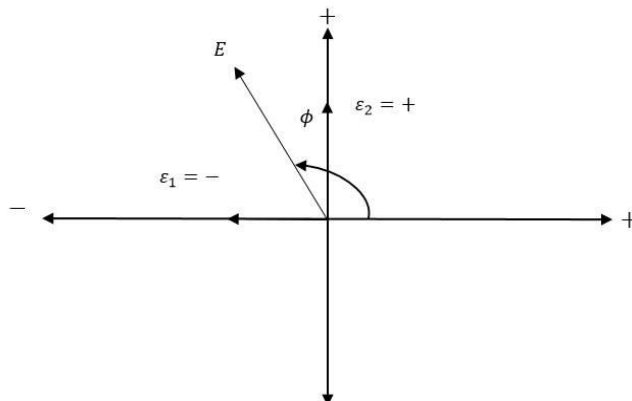
According to equation (31) the material is left handed when

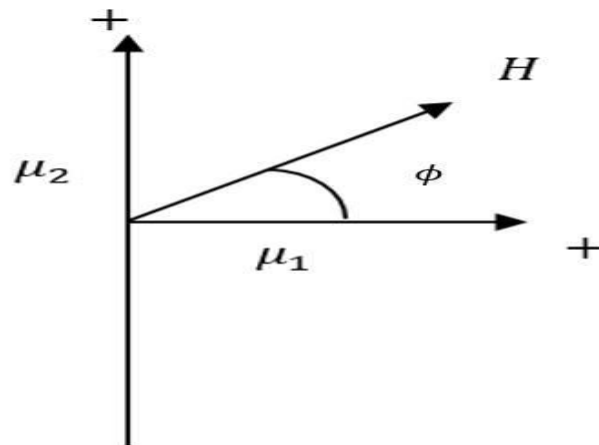
$$\mu_{1=+} \quad \epsilon_{2=+} \quad \mu_{2=+} \quad \epsilon_{1=-}$$

i.e., when

$$90^\circ < \theta < 180^\circ$$

$$0^\circ < \phi < 90^\circ$$



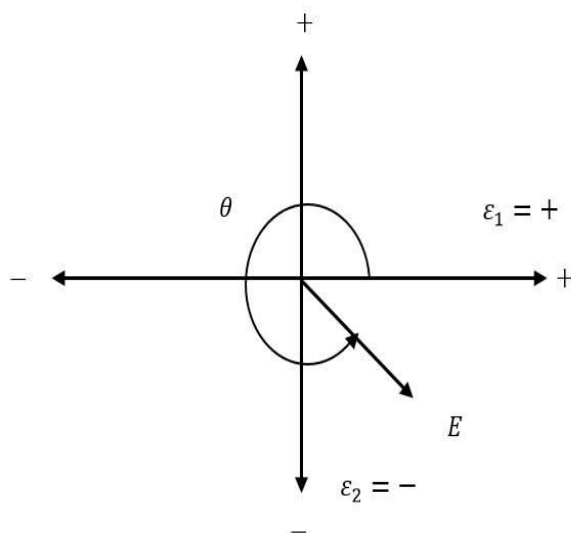
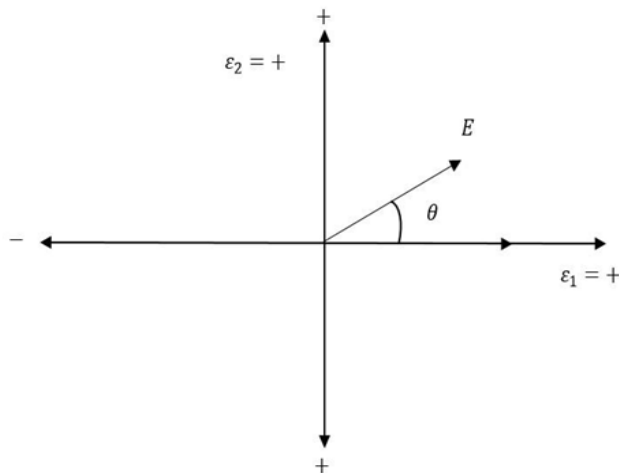


According to equation (32) the material is left handed when

$\mu_2 = -$ $\epsilon_1 = +$ $\mu_1 = +$ $\epsilon_2 = +$
 i.e., when

$$0^\circ < \theta < 90^\circ$$

$$270^\circ < \phi < 360^\circ$$



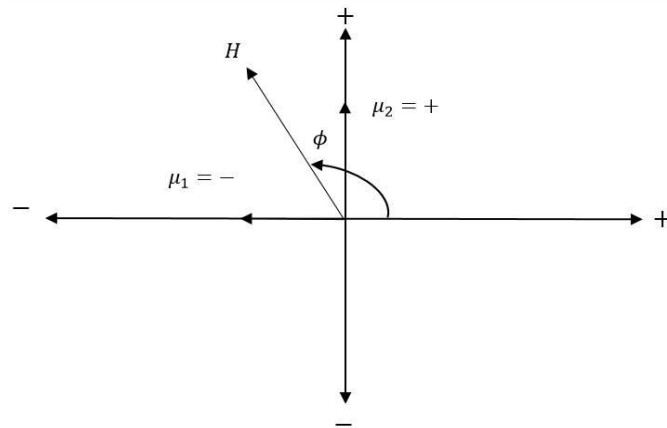
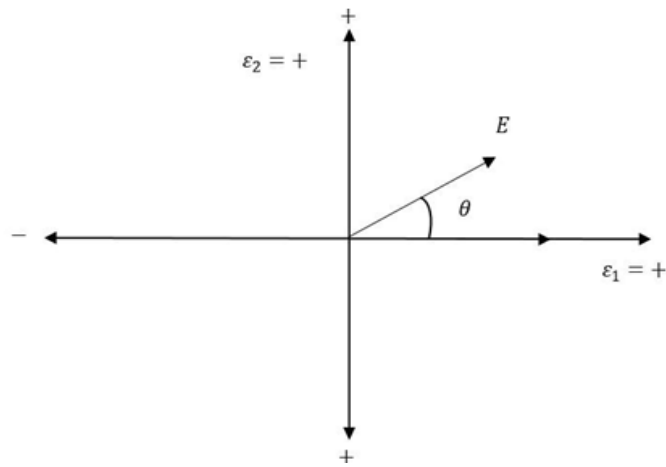
According to equation (34) the material is left handed when

$\mu_1 = -$ $\epsilon_2 = +$ $\mu_2 = +$ $\epsilon_1 = +$

i.e. ,when

$$0^\circ < \theta < 90^\circ$$

$$0^\circ < \phi < 180^\circ$$



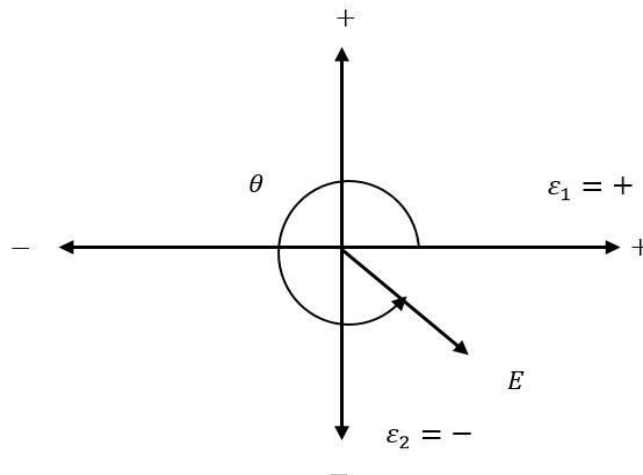
According to equation (35) the material is left handed when

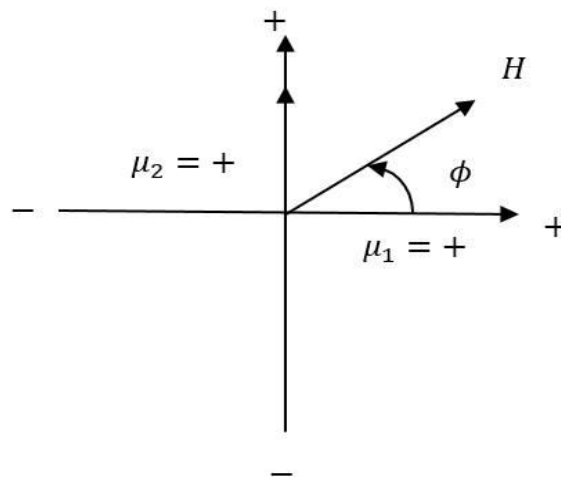
$$\mu_{1=+} \quad \epsilon_{2=-} \quad \mu_{2=+} \quad \epsilon_{1=+}$$

i.e. ,when

$$270^\circ < \theta < 360^\circ$$

$$0^\circ < \phi < 90^\circ$$





IV. CONCLUSION

The model shows that left handed materials atoms should possess some electric and magnetic properties. The atoms need to be in the form of electric and magnetic dipoles having certain orientations and values w.r.t the external fields such that the refractive index becomes negative.

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