

Mix Design of Light Weight PSA-Concrete Based On Regression Theory

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ABSTRACT

In coastal regions of Nigeria, increasing proportions of periwinkle shells are being used in building concrete houses in these localities. Currently, their use as coarse aggregates, are based on trial and error method. In this work, regression theory by Osadebe is used in developing a mix design method for prescribing mix proportions for any given compressive strength of concrete made with periwinkle shell as aggregate. Besides, the method can be used in determining the compressive strength of periwinkle shell aggregate (PSA) concrete obtainable from a specified mix proportion of constituents. Concrete specimen were produced from mixes proportioned to have different quantities of periwinkle shell, and tested in compression on the 28th. day. The experimental data generated and regression theory by Osadebe (2003), were used in developing a mix design method. The results obtained from the newly developed method agreed closely with the corresponding control experimental results. The maximum optimum compressive strength that can be designed for is 12.75N/mm² which corresponds to a mix proportion of 1: 2.04: 2.4 and water/cement ratio of 0.6.

Keywords: Osadebe's regression theory, mix design, PSA Concrete, Compression Strength.

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I. INTRODUCTION

Over the years, periwinkle shells, have been used in one form or the other. Around the coastal states and regions of Nigeria, they have been used for landscaping and land control of some sort. But only a small percentage of the periwinkle shells, is used as coarse aggregate in concrete in areas where there is neither stone nor gravel, and the procurement of such conventional aggregate, is expensive (Adewuyi and Adegoke, 2008). One of the major factors responsible for inadequate provision or acquisition of houses is high cost of building materials. For this reason, there is a tendency to use other materials such as periwinkle shell in the production of concrete.

Several tests have been conducted on lightweight solid waste materials, in order to determine their effectiveness as alternative aggregate materials, Ding et al (2004) showed that the type of aggregate well as the aggregate/binder ratio affect the compressive strength of concrete. However, tests on aggregates alone, are not effective means of predicting aggregate performance (Tony et al, 2001 cited in Ugwuibe, 2015)).

Eurolightcon (2000) stated categorically, that there is no simple relationship between the crushing resistance of lightweight aggregate and the properties of concrete. But, the crushing resistance values, can give idea of the compressive strength. In his own research, Muller (2001) showed that higher unit weight results into higher compressive strength. Earlier on, Balogun (1993), showed that the denser materials yield high compressive strengths. But, periwinkle shell, palm kernel shell-aggregate concrete are expected to yield lower compressive strengths.

In order to exploit and use to the fullest extent, the periwinkle shell as aggregate in light weight concrete, it is necessary to develop. a mix design method for determining the compressive strength of concrete

made from periwinkle shell aggregate. Besides, the mix design method developed, should be able to prescribe concrete mix ratios that can yield a periwinkle shell aggregate concrete of desired compressive strength.

Thus, the work is concerned with the development of a mix design method based on Osadebe's regression Theory, for determining the compressive strength of periwinkle shell aggregate concrete obtainable from a given mix ratio of constituents, and vice versa. The development of such a mix design method for psa-concrete would ensure the production of psa-concrete mixes whose design strengths is not compromised.

II. REGRESSION THEORY

This work is based on the regression theory developed by Osadebe (2003). The theory proposed a function, F(z) for determining the responses (properties of mixtures that are dependent on the proportion of the mixture components

The function, F(z) is continuous and differentiable with respect to its predictors, zi (Ogar, 2009). Using Taylors series, Osadebe expanded the function in the neighborhood of a chosen points

Z(o) =

Assuming the quantity of concrete is designated S, then for a concrete mixture with four components

Where S₁, S₂, S₃ and S₄ are the actual portions of the mixture components.

But, the mixture components are subject to the requirement that the sum of all the mixture components must be equal to one (Scheffe, 1958).

Dividing Equation (3) by S, gives Equation (4)

S₁/S + S₂/S + S₃/S + S₄/S = S/S (4)
 Let Z_i = S_i/S (5)

Substituting Equation (5) into Equation (4) yields Equation (6)

z₁ + z₂ + z₃ + z₄ = 1 (6)

Multiplying Equation (6) by 10 yields Equation (7).

10z₁ + 10z₂ + 10z₃ + 10z₄ = 10 (7)

Let 10z₁ = Z (8)

Therefore, Z₁ + Z₂ + Z₃ + Z₄ = 10 (9)

Choosing point z^(o) as the origin yields:

Z₁^(o) = 0, Z₂^(o) = 0, Z₃^(o) = 0 and Z₄^(o) = 0

Assuming:

(10)

(11)

(12)

(13)

Substituting Equations (10) - (13) into Equation (1) gives:

(14)

Where respectively

Multiplying Equation (6) by b_o yields Equation (15)

b_o = b_oZ₁ + b_oZ₂ + b_oZ₃ + b_oZ₄ (15)

Multiplying Equation (6) successively by Z₁, Z₂, Z₃ and Z₄, and rearranging the products, results into Equations (16), (17), (18) and (19) respectively

(16)

(17)

(18)

(19)

Substituting Equations (16) - (19) into Equation (14) and simplifying yields Equation (20)

Where

and

Equation (19) can be stated in a compact form as follows:

where

The Equations (20) and (23) are the response functions for determining any desired property of a four-component mixture, such as concrete mixture when the proportions of the components are specified. The response function can also be used for determining various proportions of PSA-concrete components required to produce PSA – concrete of a particular property.

Determination of the coefficients of the response function for the n^{th} observation point

For the n^{th} observation point, Equation (23) becomes Equation (24)

where and $n = 1, 2, 3, \dots, 10$

Since, the n^{th} observation will have Y^n response corresponding to predictors, Equation (24) can be written in a matrix form as follows:

Rearranging Equation (25) yields:

The values of the fractional portions were determined from actual mix proportions using Equation (5).

Table 1 and 2 show the values of the actual mix proportions, and their corresponding fractional portions, when and respectively (given in appendix).

The values of the fractional portions, were assembled into $Z^{(n)}$ matrix, from which the inverse matrix $Z^{(n)}$ is obtain (given in Table 3a in Appendix). The values of Y matrix are compressive strength of concrete specimen tested in the laboratory (see Table 4). With the values of $Z^{(n)}$ matrix and $Y^{(n)}$ matrix known, the values of the coefficients, of Equation (26) can be determined.

III. COMPRESSIVE STRENGTH TEST

Compressive strength tests, were carried out in order to generate the responses, Y_i , needed to determine and to verify the final response function F . A total of twenty mix ratios were used to produce sixty periwinkle shell-concrete cubes, each measuring 150x150 x150mm in size. The psa-concrete were cured for 28 days and tested in a universal testing machine thereafter. The compressive test results are given in Table 4 (given in Appendix).

Thirty of the test results were used to determine the final response function, while the remaining test results, were used to verify the adequacy of the formulated response function.

IV. RESULTS AND DISCUSSION

The results of the compressive strength tests of the sixty PSA - concrete cubes, are given in Table 4.

The mean values of the responses, is obtained from Equation (27)

And the values of the variances of the replicates, is determined from Equation (28)

where

Y_i = responses

= Mean of responses

n = number of observations of every ports

N = number of design points

Total variance of replicates,

$$\begin{aligned}
 &= 21 - 981/(20-1) \\
 &= 1.1569
 \end{aligned}
 \tag{29}$$

Therefore, standard error of the replicates, S_y is obtain as follows:

$$\tag{30}$$

This value is used to determine the t-statistics value for the regression function.

Determination of the Final Regression Function.

Substituting the PSA – concrete strengths from the Laboratory tests (given in Table 4) into Equation (26), yields the coefficients of the regression function

Substituting these coefficients, , into Equation (20), gives the final regression function, Y.

The Equation (31) is the final response function, Y, for determining the compressive strength of periwinkle shell aggregate concrete when the mix proportions are specified.. It can also be used to determine the mix proportions when the compressive strength verification of adequacy of the response function.

The adequacy of the response function was verified using student’s t-test and controlled experimental results presented in Table 5.

The test was carried out at a significance level, α of 0.05.

Table 5: Computations of statistical t – test for Osadebe regression theory

S/N	Y_E	Y_M	$D_i = Y_E - Y_M$	$D_A - D_i$	$(D_A - D_i)^2$
C1	9.67	9.903	-0.233	0.2667	0.07112889
C2	9.43	9.957	-0.527	0.5607	0.31438449
C3	9.78	10.031	-0.251	0.2847	0.08105409
C4	9.49	9.827	-0.337	0.3707	0.13741849
C5	10.33	9.991	0.339	-0.3053	0.09320809
C6	8.56	8.093	0.467	-0.4333	0.18774889
C7	9.29	9.295	-0.005	0.0387	0.00149769
C8	10.193	10.103	0.09	-0.0563	0.00316969
C9	11.17	10.716	0.454	-0.4203	0.17665209
C10	8.91	8.57	0.34	-0.3063	0.09381969

Substitution of the relevant values given, into the following equations, yields:

$$\tag{32}$$

$$= 0.128898011$$

(34)

$$= 0.359023692$$

(35)

$$= 0.0337 (10)^{1/2} / 0.359023692$$

$$= 1.833$$

This t-value is greater than t-value of 0.297. Therefore, the differences between the Laboratory results and the predicted results, are not significant. Consequently, the regression function can be used in predicting the mix proportions of PSA - light weight concrete of given compressive strength and vice versa.

Table 6 Comparison of Experimental and Predicted Results

S/N	Experimental Test Results (Y_E) N/mm ²	Predicted Results (Y_p) N/mm ²	Difference ($Y_E - Y_p$)	% Difference ($(Y_E - Y_p) / Y_E$)
1	9.67	9.90	0.23	2.48
2	9.43	9.96	0.53	5.62
3	9.78	10.03	0.25	2.56
4	9.49	9.83	0.36	3.79
5	10.33	9.99	0.34	3.29
6	8.56	8.09	0.47	5.49
7	9.29	9.30	0.01	0.00
8	10.19	10.10	0.09	0.01
9	11.17	10.72	0.45	0.01
10	8.91	8.57	0.34	3.81

The compressive strengths of PSA light weight concrete obtained from laboratory tests were compared with the compressive strengths obtained from the newly formulated regression function. There was close agreement between these results with the maximum difference being 5.62 percent. And so, the newly formulated regression function can be used accurately In the design of PSA - light weight concrete.

V. CONCLUSION

The development of a mix design method is a critical factor in the production and use of cheap and affordable building materials from wastes. Osadebes regression theory has been used successively to develop a new mix design method for PSA - light weight concrete. The results obtained from the newly formulated method of designing PSA - light weight concrete compared favourable with experimental results. The use of the newly formulated mix design method, will ensure the production of PSA – concrete with the desired strength.

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APPENDICES

Table 1: Values of the actual mixture proportions and their corresponding fractional

POINT	S ₁	S ₂	S ₃	S ₄	S	Z ₁	Z ₂	Z ₃	Z ₄	Z
N1	0.54	1	2.2	2.75	6.49	0.08320493	0.154083205	0.33898305	0.423728814	1
N2	0.74	1	2	2.35	6.09	0.12151067	0.164203612	0.32840722	0.385878489	1
N3	0.84	1	1.8	1.95	5.59	0.15026834	0.178890877	0.32200358	0.348837209	1
N4	0.94	1	1.6	1.6	5.14	0.18287938	0.194552529	0.31128405	0.311284047	1
N12	0.64	1	2.1	2.55	6.29	0.10174881	0.158982512	0.33386328	0.405405405	1
N13	0.69	1	2	2.35	6.04	0.11423841	0.165562914	0.33112583	0.389072848	1
N14	0.74	1	1.9	2.175	5.815	0.12725709	0.171969046	0.32674119	0.374032674	1
N23	0.79	1	1.9	2.15	5.84	0.13527397	0.171232877	0.32534247	0.368150685	1
N24	0.84	1	1.8	1.975	5.615	0.14959929	0.17809439	0.3205699	0.35173642	1
N34	0.89	1	1.7	1.775	5.365	0.16589003	0.18639329	0.31686859	0.330848089	1
C1	0.765	1	1.9	2.1625	5.8275	0.13127413	0.171600172	0.32604033	0.371085371	1
C2	0.86	1	1.74	1.855	5.455	0.15765353	0.183318057	0.31897342	0.340054995	1
C3	0.73	1	1.96	2.275	5.965	0.12238055	0.167644593	0.385834	0.38139145	1
C4	0.8	1	1.84	2.05	5.69	0.14059754	0.175746924	0.32337434	0.360281195	1
C5	0.72	1	1.96	2.28	5.96	0.12080537	0.167785335	0.32885906	0.382550336	1
C6	0.66	1	2.06	2.475	6.195	0.10653753	0.1614205	0.33252623	0.399515738	1
C7	0.75	1	1.92	2.205	5.875	0.12765957	0.170212766	0.32680851	0.375319149	1
C8	0.75	1	1.9	2.17	5.82	0.12886598	0.171821306	0.32646048	0.372852234	1
C9	0.77	1	1.86	2.095	5.725	0.13449782	0.174672489	0.32489083	0.365938865	1
C10	0.68	1	2.02	2.4	6.1	0.11147541	0.163934426	0.33114754	0.393442623	1

Table 2: Values of the Actual Mixture Proportions and the Corresponding Fractional Portions for a System of ΣZ = 10

POINT	Z ₁	Z ₂	Z ₃	Z ₄	Z ₁ Z ₂	Z ₁ Z ₃	Z ₁ Z ₄	Z ₂ Z ₃	Z ₂ Z ₄	Z ₃ Z ₄
N1	0.08	0.1541	0.339	0.42373	0.0128205	0.02821	0.03525633	0.052231595	0.06528949	0.143636886
N2	0.12	0.1642	0.328	0.38588	0.0199525	0.0399	0.04688836	0.053925653	0.06336264	0.126725284
N3	0.15	0.1789	0.322	0.34884	0.0268816	0.04839	0.05241919	0.057603502	0.06240379	0.112326829
N4	0.18	0.1946	0.311	0.31128	0.0355796	0.05693	0.05692743	0.060561099	0.0605611	0.096897758
N5	0.10	0.159	0.334	0.40541	0.0161763	0.03397	0.04124952	0.05307822	0.06445237	0.135349976
N6	0.11	0.1656	0.331	0.38907	0.0189136	0.03783	0.04444706	0.054822157	0.06441603	0.128832069
N7	0.13	0.172	0.327	0.37403	0.0218843	0.04158	0.04759831	0.05618937	0.06432204	0.12221188
N8	0.14	0.1712	0.325	0.36815	0.0231634	0.04401	0.04980121	0.055709326	0.0630395	0.119755052
N9	0.15	0.1781	0.321	0.35174	0.0266428	0.04796	0.05261952	0.057091701	0.06264228	0.11275611
N10	0.17	0.1864	0.317	0.33085	0.0309208	0.05257	0.0548844	0.059062179	0.06166786	0.104835369
C1	0.13	0.1716	0.326	0.37109	0.0225267	0.0428	0.04871391	0.055948576	0.06367831	0.120988795
C2	0.16	0.1833	0.319	0.34005	0.0289007	0.05029	0.05361087	0.058473587	0.06233822	0.108468504
C3	0.12	0.1676	0.329	0.38139	0.0205164	0.04021	0.0466749	0.055085231	0.06393821	0.125318901
C4	0.14	0.1757	0.323	0.36028	0.0247096	0.04547	0.05065465	0.056832046	0.06331831	0.116505694
C5	0.12	0.1678	0.329	0.38255	0.0202694	0.03973	0.04621413	0.055177695	0.0641863	0.125805144
C6	0.11	0.1614	0.333	0.39952	0.0171973	0.03543	0.04256342	0.053676551	0.06449003	0.132849463
C7	0.13	0.1702	0.327	0.37532	0.0217293	0.04172	0.04791308	0.055626981	0.06388411	0.122657492
C8	0.13	0.1718	0.326	0.37285	0.0221419	0.04207	0.04804797	0.056092866	0.06406396	0.12172152
C9	0.13	0.1747	0.325	0.36594	0.0234931	0.0437	0.04921798	0.05674949	0.06391945	0.118890181
C10	0.11	0.1639	0.331	0.39344	0.0182747	0.03691	0.04385918	0.054286482	0.06449879	0.130287557

Table 3a: Z⁽ⁿ⁾ Matrix

0.08	0.1541	0.339	0.42373	0.01282	0.02821	0.03526	0.052232	0.065289	0.143637
0.12	0.1642	0.328	0.38588	0.019952	0.03991	0.046888	0.053926	0.063363	0.126725
0.15	0.1789	0.322	0.34884	0.026882	0.04839	0.052419	0.057604	0.062404	0.112327
0.18	0.1946	0.311	0.31128	0.03558	0.05693	0.056927	0.060561	0.060561	0.096898
0.10	0.159	0.334	0.40541	0.016176	0.03397	0.04125	0.053078	0.064452	0.13535

0.11	0.1656	0.331	0.38907	0.018914	0.03783	0.044447	0.054822	0.064416	0.128832
0.13	0.172	0.327	0.37403	0.021884	0.04158	0.047598	0.056189	0.064322	0.122212
0.14	0.1712	0.325	0.36815	0.023163	0.04401	0.049801	0.055709	0.06304	0.119775
0.15	0.1781	0.321	0.35174	0.026643	0.04796	0.05262	0.057092	0.062642	0.112756
0.17	0.1864	0.317	0.33085	0.030921	0.05257	0.054884	0.059062	0.061668	0.104835

Table 3b: Inverse of Z⁽ⁿ⁾ Matrix

.981	22046.841	-38893.615	-12025.373	-33805.66	21877.973	-3854.291	9058.356	-18970.820	46791.596
.393	-30567.282	-190681.291	-36775.050	-28886.314	-6912.384	17262.312	211742.404	-	177365.226
.807	-92788.602	-663656.544	-210214.987	-9574.868	189168.821	-	395839.685	-	743085.416
.215	-17716.690	-225344.049	-76073.100	-11443.542	76098.071	121663.711	110996.678	-201408.605	262573.866
.870	-42502.616	143382.886	9986.877	125175.811	41149.298	-46292.943	-211826.533	-63667.139	-117363.888
.243	47707.906	952985.924	315963.654	87569.583	-364934.448	-71767.846	-440354.552	17040.052	-1100613.678
.186	23827.427	11170.669	32077.308	1649.310	-4421.809	211312.635	-99058.609	257469.992	-118515.685
.178	303945.895	1518553.849	426946.276	-5848.315	261393.874	4243.647	1177851.820	577753.425	-1601188.325
.449	-64176.007	38644.884	2270.219	84008.027	-13283.611	-24438.164	-21111.099	63668.983	-43014.719
.296	198485.295	1661084.701	539843.386	42093.860	-506022.222	320080.147	-926411.267	487585.885	-1888150.593

Table 4: 28th Day Experimental Compressive Strength Test Results of PSA-Concrete

S/N	Points of Observation	Average Cube Strength (N/mm ²)
1	N ₁	11.35
2	N ₂	11.66
3	N ₃	9.66
4	N ₄	10.24
5	N ₅	7.70
6	N ₆	12.65
7	N ₇	10.59
8	N ₈	10.97
9	N ₉	10.21
10	N ₁₀	9.26
11	C ₁	9.67
12	C ₂	9.43
13	C ₃	9.78
14	C ₄	9.49
15	C ₅	10.33
16	C ₆	8.56
17	C ₇	9.29
18	C ₈	10.19
19	C ₉	11.17
20	C ₁₀	8.91