

Determination of the spatial components of the magnetic induction created by a coil with a circular arc head

J. VELONTSOA^{1,3}, R. Saray RAHARINAIVO^{1,2}, A. ATTOUMANI³, U. Michaël MAHAVELONA³, B. KALL²

¹Direction of the School of the Industrial engineering, Higher Institute of Technology of Antsiranana

²Doctoral School set of themes " Ergonomics, Nature and Structure of the Matter " of the University of Antsiranana.

³Doctoral School set of themes " Renewable Energies and Environment " of the University of Antsiranana. BPO, 201 Antsiranana, Madagascar, E-mail: velontsoajeannot@gmail.com

ABSTRACT— : This article is dedicated to the calculation of the spatial components of the magnetic induction generated by a loop with a circular arc head, commonly found in the rotor of a wound-rotor asynchronous machine. The calculation is based on the Biot - Savart law. The expressions are derived for a vacuum environment. For a medium with permeability μ , the results must be multiplied by this permeability. To obtain the induction for a coil, the result for a single loop is multiplied by the number of loops comprising the coil. The results are verified using curves, with each component validated through three different curves, using the dimensions X, Y, and Z as variables.

Keywords: loop, spatial component, machine, magnetic induction, coil, rotor, Biot-Savart.

Date of Submission: 03-02-2025

Date of acceptance: 14-02-2025

I. INTRODUCTION

Since its invention until today, the rotating electrical machine [1] has continued to provide researchers [2] with innovative ideas leading to improvements or developments of models [3]. Indeed, competition arises in terms of quality and the lifespan of machines, in accordance with different standards [4]. Minimizing losses in machines remains the major challenge of current technology [5]. This challenge comes up against the consequences of inevitable movements of electric charges in the magnetic circuit of the machine during its operation. Indeed, at any point in space, the magnetic excitation field is described by the spatio-temporally variable vector \vec{H} [6-7]. Associated with this field is the induction which induces the voltage at the terminals of the coils or in conductive domains by causing losses. Taking into account electromagnetic compatibility requires knowledge of the components of this field.

It is the set of vectors \vec{H} that constitutes the magnetic excitation field (vector field). In practice, iron filings (detector), sprinkled in the vicinity of the source (electric machine powered for example), allows to visualize (magnetic spectrum) the excitation field [8]. The authors of this article go beyond the design domain and analyze the presence of the induction component outside the design domain so that everyone becomes aware of it by aiming for better electromagnetic compatibility. Their work will be limited to the search for the spatial components of the magnetic induction generated by a circular head coil or a circular end coil with a pseudo-rectangular body.

II. METHODOLOGIE

II.1 Theoretical basis

The study is based on the Biot and Savart law and uses the following relation (1):

$$\vec{B}_M = \frac{\mu I}{4\pi} \int_{\text{ligne}} \vec{dl} \wedge \frac{\vec{PM}}{\|\vec{PM}\|^3} \quad (1)$$

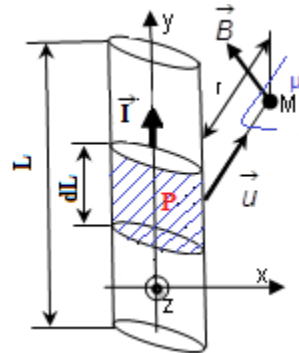


Fig 1: Element of a conductor carrying a current creating an induction at a point in space

II.2 Description du système étudié

Figure 1 below shows a coil whose heads are two convex arcs AB and CD of radius R, with respective centers O₁ and O₂, connected by two parallel conductors AD and BC of length 2b. The calculation reference has as its origin O, the middle of [O₁ and O₂]. The two arcs and the two conductors are thus symmetrical with respect to the Oz axis as shown in Fig.2 below and form a magnetic pole. An electric machine has 2p poles and the angle of the arcs is $\theta_{pole} = \frac{\pi}{p}$.

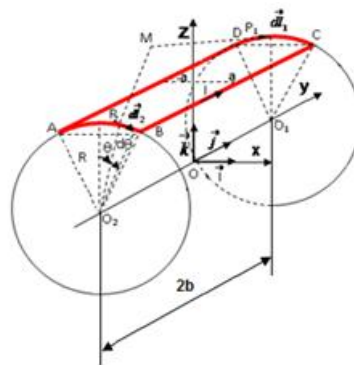


Fig. 2. Circular arc head coil carrying a current I

II.2 Determination of the spatial components of induction

The induction at any point M such that $\vec{OM} = X\vec{i} + Y\vec{j} + Z\vec{k}$ is generated by two parallel conductors AD and BC and by the two AB and CD.

Let us suppose a point $Q_1\left(R\sin\left(\frac{\pi}{2p}\right); -x; R\cos\left(\frac{\pi}{2p}\right)\right)$ of the bar [AD] and a point $Q_2\left(-R\sin\left(\frac{\pi}{2p}\right); x; R\cos\left(\frac{\pi}{2p}\right)\right)$ of the bar [BC] symmetrical with respect to the center of the parallelogram

ABCD, around these points we have the current elements $I dy \vec{j}$ and $-I dy \vec{j}$. The elementary inductions at M are expressed:

$$\vec{dB}_{Q1} = \frac{\mu_0 I dy \vec{j} \wedge \vec{Q_1M}}{4\pi \|\vec{Q_1M}\|^3} \tag{2}$$

$$\vec{dB}_{Q2} = -\frac{\mu_0 I dy \vec{j} \wedge \vec{Q_2M}}{4\pi \|\vec{Q_2M}\|^3} \tag{3}$$

Comme

$$\vec{Q_1M} = \left(X - R\sin\frac{\pi}{2p}\right)\vec{i} + (Y + y)\vec{j} + \left(Z - R\cos\frac{\pi}{2p}\right)\vec{k} \tag{4}$$

$$\vec{Q}_2\vec{M} = \left(X + R\sin\frac{\pi}{2p} \right) \vec{i} + (Y-y) \vec{j} + \left(Z - R\cos\frac{\pi}{2p} \right) \vec{k} \tag{5}$$

In this case the expressions of the elementary inductions become:

$$\vec{dB}_{Q1} = \frac{\mu_0 I}{4\pi} \frac{\left[\left(Z - R\cos\frac{\pi}{2p} \right) \vec{i} - \left(X - R\sin\frac{\pi}{2p} \right) \vec{k} \right]}{\left[\left(X - R\sin\frac{\pi}{2p} \right)^2 + (Y+y)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2 \right]^{\frac{3}{2}}} dy \tag{6}$$

$$\vec{dB}_{Q2} = \frac{\mu_0 I}{4\pi} \frac{\left[-\left(Z - R\cos\frac{\pi}{2p} \right) \vec{i} + \left(X + R\sin\frac{\pi}{2p} \right) \vec{k} \right]}{\left[\left(X + R\sin\frac{\pi}{2p} \right)^2 + (Y-y)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2 \right]^{\frac{3}{2}}} dy \tag{7}$$

We pose

$$\rho_1 = \int_{-b}^b \frac{dy}{\left[\left(X - R\sin\frac{\pi}{2p} \right)^2 + (Y+y)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2 \right]^{\frac{3}{2}}} \tag{8}$$

$$\rho_2 = \int_{-b}^b \frac{dy}{\left[\left(X + R\sin\frac{\pi}{2p} \right)^2 + (Y-y)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2 \right]^{\frac{3}{2}}} \tag{9}$$

As $-b \leq y \leq b$, the total induction is expressed as follows

$$\vec{B}_{\text{barre}}(M) = \frac{\mu_0 I}{4\pi} \left[\left(Z - R\cos\left(\frac{\pi}{2p}\right) \right) (\rho_1 - \rho_2) \vec{i} + \left(\rho_2 \left(X + R\sin\left(\frac{\pi}{2p}\right) \right) - \rho_1 \left(X - R\sin\left(\frac{\pi}{2p}\right) \right) \right) \vec{k} \right] \tag{10}$$

With

$$\rho_1 = \frac{1}{\left(X - R\sin\left(\frac{\pi}{2p}\right) \right)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2} \left[\frac{Y+b}{\sqrt{\left(X - R\sin\left(\frac{\pi}{2p}\right) \right)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2 + (Y+b)^2}} - \frac{Y-b}{\sqrt{\left(X - R\sin\left(\frac{\pi}{2p}\right) \right)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2 + (Y-b)^2}} \right] \tag{11}$$

$$\rho_2 = \frac{1}{\left(X + R\sin\left(\frac{\pi}{2p}\right) \right)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2} \left[\frac{Y+b}{\sqrt{\left(X + R\sin\left(\frac{\pi}{2p}\right) \right)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2 + (Y+b)^2}} - \frac{Y-b}{\sqrt{\left(X + R\sin\left(\frac{\pi}{2p}\right) \right)^2 + \left(Z - R\cos\frac{\pi}{2p} \right)^2 + (Y-b)^2}} \right] \tag{12}$$

The two arcs AB and CD do not belong to the same plane as the two parallel sides. Let P_1 and P_2 be two points taken respectively CD and AB such that $P_1(R\sin\theta; -b; R\cos\theta)$ and $P_2(-R\sin\theta; b; R\cos\theta)$. The current elements taken around P_1 and P_2 give the following elementary inductions:

$$\vec{dB}_{P1} = \frac{\mu_0 I}{4\pi} \frac{R d\theta \vec{e}_0 \wedge \vec{P}_1\vec{M}}{\left\| \vec{P}_1\vec{M} \right\|^3} \tag{13}$$

$$\vec{dB}_{P2} = \frac{\mu_0 I}{4\pi} \frac{R d\theta \vec{e}_0' \wedge \vec{P}_2\vec{M}}{\left\| \vec{P}_2\vec{M} \right\|^3} \tag{14}$$

In P_1

$$\vec{e}_0 = \cos\theta \vec{i} - \sin\theta \vec{k} \tag{15}$$

In P_2

$$\vec{e}_0' = -\cos\theta \vec{i} - \sin\theta \vec{k} \tag{16}$$

Alors on a :

$$\vec{P}_1\vec{M} = (X-R\sin\theta)\vec{i} + (Y+b)\vec{j} + (Z-R\cos\theta)\vec{k} \tag{17}$$

$$\vec{P}_2\vec{M} = (X+R\sin\theta)\vec{i} + (Y-b)\vec{j} + (Z-R\cos\theta)\vec{k} \tag{18}$$

$$\vec{e}_0 \wedge \vec{P}_1\vec{M} = (Y+b)\cos\theta\vec{k} - (Z\cos\theta + X\sin\theta - R)\vec{j} + (Y+b)\sin\theta\vec{i} \tag{19}$$

$$\vec{e}'_0 \wedge \vec{P}_2\vec{M} = -(Y-b)\cos\theta\vec{k} + (Z\cos\theta - X\sin\theta - R)\vec{j} + (Y-b)\sin\theta\vec{i} \tag{20}$$

In this case we have:

$$\vec{dB}_{P1} = \frac{\mu_0 I}{4\pi} \frac{Rd\theta \left[(Y+b)\sin\theta\vec{i} - (Z\cos\theta + X\sin\theta - R)\vec{j} + (Y+b)\cos\theta\vec{k} \right]}{\left[(X-R\sin\theta)^2 + (Y+b)^2 + (Z-R\cos\theta)^2 \right]^{\frac{3}{2}}} \tag{21}$$

$$\vec{dB}_{P2} = \frac{\mu_0 I}{4\pi} \frac{Rd\theta \left[(Y-b)\sin\theta\vec{i} + (Z\cos\theta - X\sin\theta - R)\vec{j} - (Y-b)\cos\theta\vec{k} \right]}{\left[(X+R\sin\theta)^2 + (Y-b)^2 + (Z-R\cos\theta)^2 \right]^{\frac{3}{2}}} \tag{22}$$

For this circular arc head coil, the magnetic induction vector in M is characterized by:

- ❖ its component along the abscissa axis B_x ;
- ❖ its component along the ordinate axis B_y ;
- ❖ its component along the side B_z ;

these components are calculated below.

II.2.1 Calculation of the radial component B_x

From relations (13) and (14), we can deduce the following elementary induction:

$$\vec{dB}_{px} = \frac{\mu_0 I}{4\pi} R\sin\theta \left[\frac{Y+b}{\left[(X-R\sin\theta)^2 + (Y+b)^2 + (Z-R\cos\theta)^2 \right]^{\frac{3}{2}}} + \frac{Y-b}{\left[(X+R\sin\theta)^2 + (Y-b)^2 + (Z-R\cos\theta)^2 \right]^{\frac{3}{2}}} \right] \vec{i} \tag{23}$$

By integrating relation (15) for $-\frac{\pi}{2p} \leq \theta \leq \frac{\pi}{2p}$, we obtain the following magnetic induction vector:

$$\vec{B}_{px} = \frac{\mu_0 I}{4\pi} R (\lambda_2 + \lambda_1) \vec{i} \tag{24}$$

the magnetic induction vector along the total (ox) axis is as follows:

$$\vec{B}_x = \frac{\mu_0 I}{4\pi} \left(R (\lambda_2 + \lambda_1) + \left(Z - R\cos\frac{\pi}{2p} \right) (\rho_1 - \rho_2) \right) \vec{i} \tag{25}$$

Let's ask

$$\lambda_1 = \int_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} \frac{(Y+b)\sin\theta}{\left[(X-R\sin\theta)^2 + (Y+b)^2 + (Z-R\cos\theta)^2 \right]^{\frac{3}{2}}} d\theta \tag{26}$$

$$\lambda_2 = \int_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} \frac{(Y-b)\sin\theta}{\left[(X+R\sin\theta)^2 + (Y-b)^2 + (Z-R\cos\theta)^2 \right]^{\frac{3}{2}}} d\theta \tag{27}$$

II.2.2 Calculation of the component along the (oy) axis B_y

From relations (13) and (14), we can deduce the following elementary induction:

$$\vec{dB}_{py} = \frac{\mu_0 I}{4\pi} R \left[\frac{Z \cos \theta + X \sin \theta - R}{\left[(X - R \sin \theta)^2 + (Y + b)^2 + (Z - R \cos \theta)^2 \right]^{\frac{3}{2}}} + \frac{Z \cos \theta - X \sin \theta - R}{\left[(X + R \sin \theta)^2 + (Y - b)^2 + (Z - R \cos \theta)^2 \right]^{\frac{3}{2}}} \right] \vec{j} \quad (28)$$

By integrating relation (20) for $-\frac{\pi}{2p} \leq \theta \leq \frac{\pi}{2p}$, we obtain the magnetic induction vector along the (oy) axis:

$$\vec{B}_y = \frac{\mu_0 I}{4\pi} R (-\lambda_2 + \lambda_3) \vec{j} \quad (29)$$

With

$$\lambda_2 = \int_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} \frac{Z \cos \theta + X \sin \theta - R}{\left[(X - R \sin \theta)^2 + (Y + b)^2 + (Z - R \cos \theta)^2 \right]^{\frac{3}{2}}} d\theta \quad (30)$$

$$\lambda_3 = \int_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} \frac{Z \cos \theta - X \sin \theta - R}{\left[(X + R \sin \theta)^2 + (Y - b)^2 + (Z - R \cos \theta)^2 \right]^{\frac{3}{2}}} d\theta \quad (31)$$

II.2.3 Calculation of the component along the axis (oz) B_z

From relations (13) and (14), we can deduce the following elementary induction:

$$\vec{dB}_{pz} = \frac{\mu_0 I}{4\pi} R \cos \theta \left[\frac{Y + b}{\left[(X - R \sin \theta)^2 + (Y + b)^2 + (Z - R \cos \theta)^2 \right]^{\frac{3}{2}}} - \frac{Y - b}{\left[(X + R \sin \theta)^2 + (Y - b)^2 + (Z - R \cos \theta)^2 \right]^{\frac{3}{2}}} \right] \vec{k} \quad (32)$$

By integrating relation (15) for $-\frac{\pi}{2p} \leq \theta \leq \frac{\pi}{2p}$, we obtain the following magnetic induction vector:

$$\vec{B}_{pz} = \frac{\mu_0 I}{4\pi} R (\lambda_3 - \lambda_4) \vec{k} \quad (33)$$

The total magnetic induction vector along the (oz) axis is as follows:

$$\vec{B}_z = \frac{\mu_0 I}{4\pi} \left(R (\lambda_3 - \lambda_4) + \left(X + R \sin \frac{\pi}{2p} \right) \rho_2 - \left(X - R \sin \frac{\pi}{2p} \right) \rho_1 \right) \vec{k} \quad (34)$$

With

$$\lambda_3 = \int_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} \frac{(Y + b) \cos \theta}{\left[(X - R \sin \theta)^2 + (Y + b)^2 + (Z - R \cos \theta)^2 \right]^{\frac{3}{2}}} d\theta \quad (35)$$

$$\lambda_4 = \int_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} \frac{(Y - b) \cos \theta}{\left[(X + R \sin \theta)^2 + (Y - b)^2 + (Z - R \cos \theta)^2 \right]^{\frac{3}{2}}} d\theta \quad (36)$$

III. RESULTATS

III.1 Component along (ox) B_x

Here are the curves of the magnetic induction component along the (ox) axis for a circular arc coil with a height of 120 mm, a radius of 100 mm, and three pole pairs, carrying a current of 3 A.

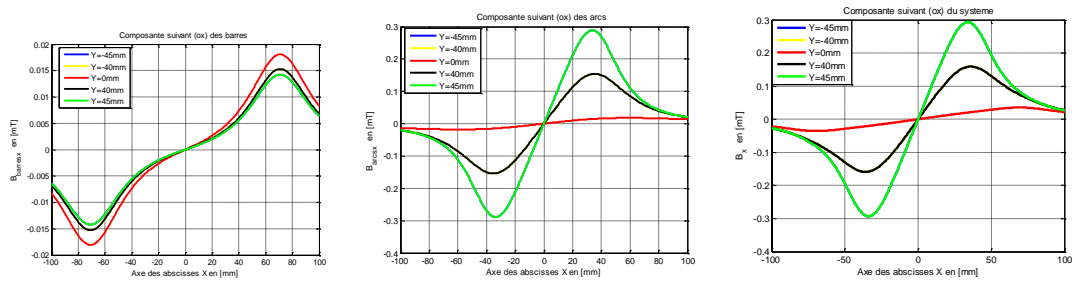


Fig. 3. Curve of a magnetic induction of the component along (ox) of the circular arc head coil by varying X for Z=R

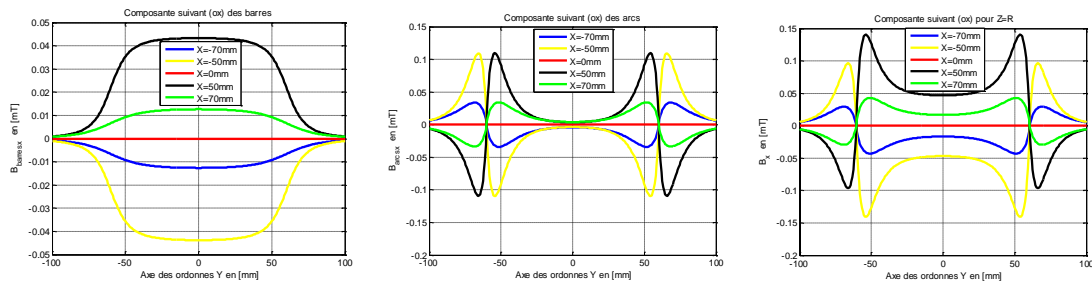


Fig. 4. Curve of a magnetic induction of the component along (ox) of the circular arc head coil by varying Y for Z=R

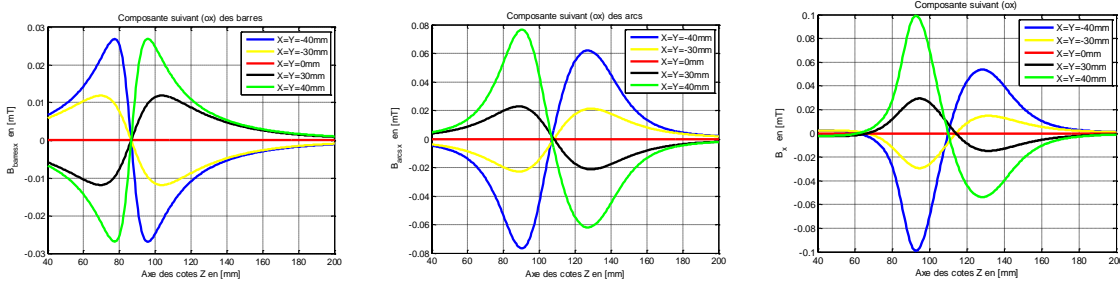


Fig. 5. Curve of a magnetic induction of the component along (ox) of the circular arc head coil while varying Z

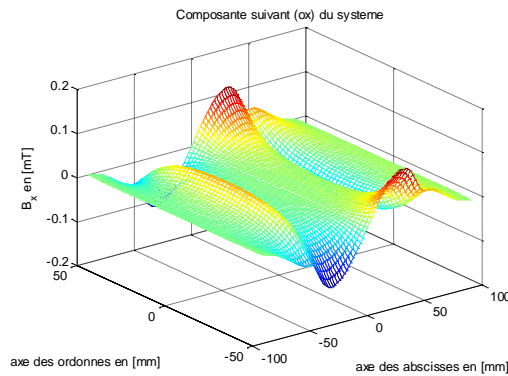


Fig. 6. Surface curve of a magnetic induction of the component along (ox) of the circular arc head coil

III.2 Component along (oy) B_y

Shown here are the tangential component of the magnetic induction for a circular arc coil with a length of 120 mm, a radius of 50 mm, and three pole pairs, carrying a current of 3 A.

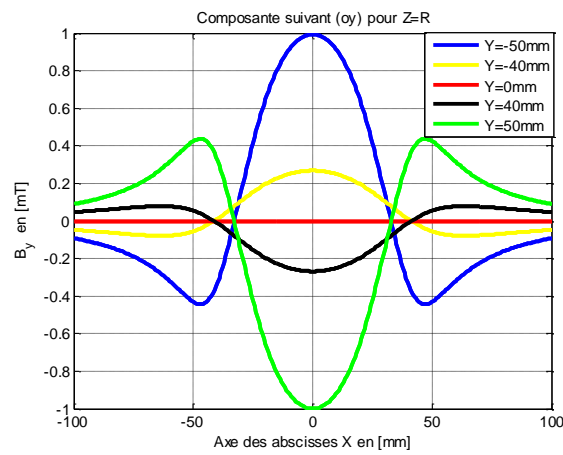


Fig. 7. Curve of a magnetic induction of the component along (oy) of the circular arc head coil by varying X for $Z=R$

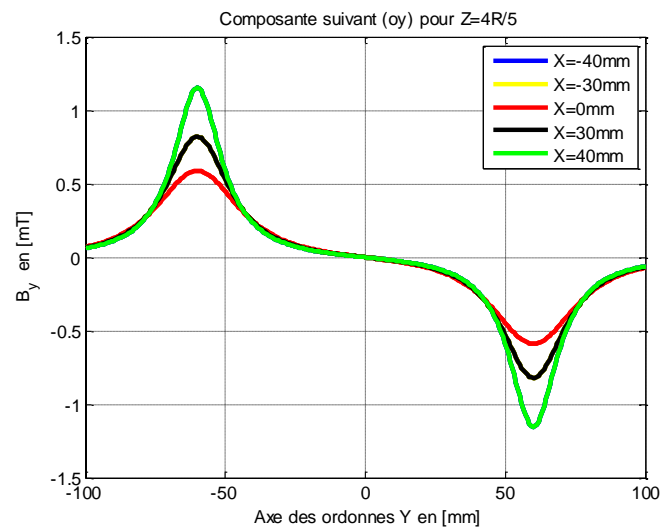


Fig. 8. Curve of a magnetic induction of the component along (oy) of the circular arc head coil varying Y for $Z=4R/5$

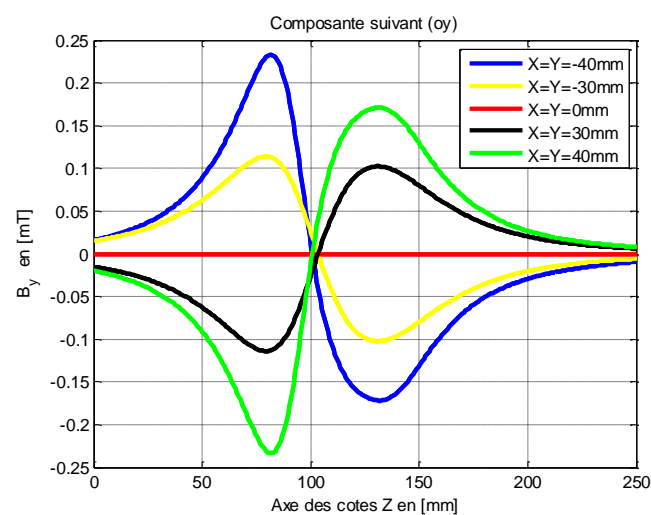


Fig. 9. Curve of a magnetic induction of the component along (oy) of the circular arc head coil while varying Z

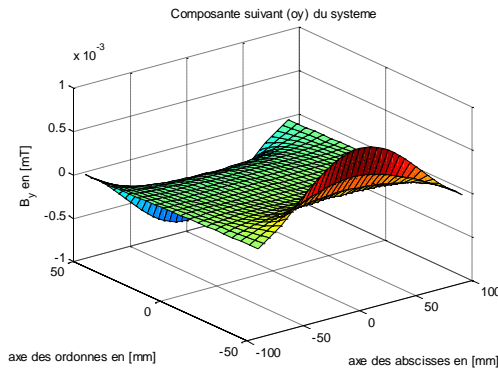


Fig. 10. Surface curve of a magnetic induction of the component along (oy) of the circular arc head coil

III.3 Component along (oz) B_z

Displayed here are the curves of the axial component of the magnetic induction for a circular arc coil with a length of 120 mm, a radius of 50 mm, and three pole pairs, carrying a current of 3 A.

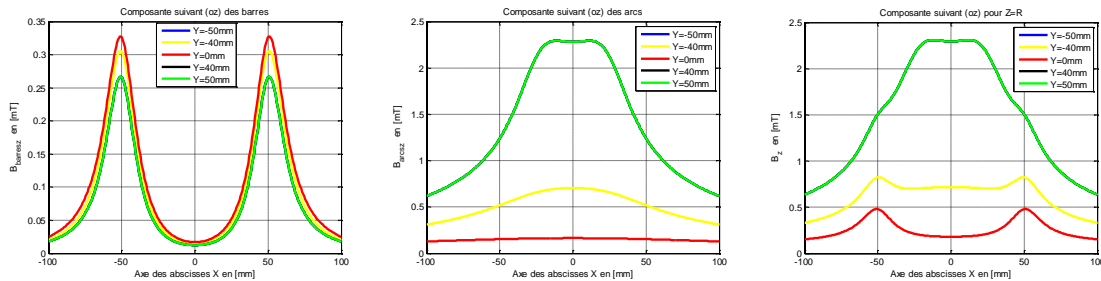


Fig. 11. Curve of a magnetic induction of the component following (oz) of the circular arc head coil by varying Z

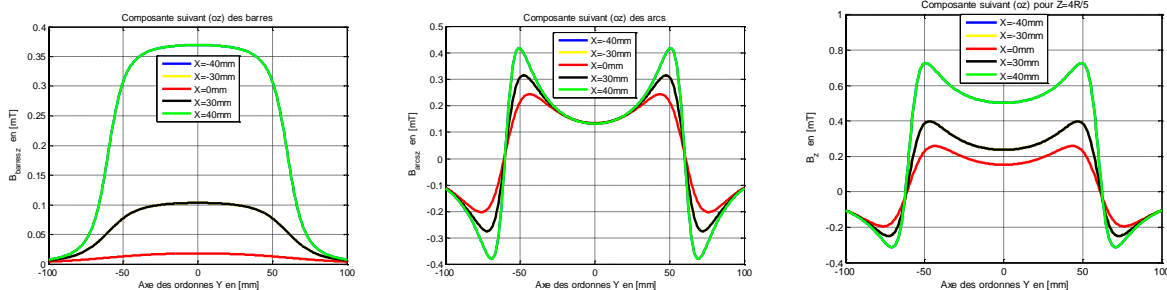


Fig.12. Curve of a magnetic induction of the component following (oz) of the circular arc head coil by varying Y for Z=4R/5.

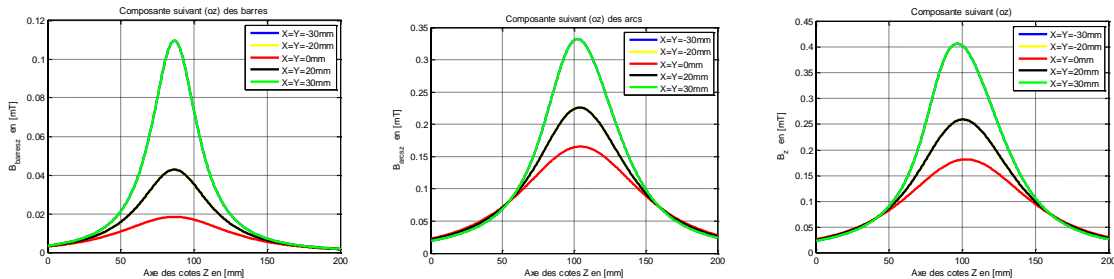


Fig. 13. Curve of a magnetic induction of the component following (oz) of the circular arc head coil by varying Z

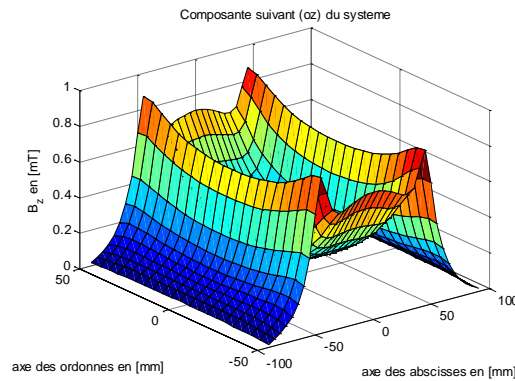


Fig. 14. Surface curve of a magnetic induction of the component along (oz) of the circular arc head coil

III.4 Ratio of components

Shown here are the curves of the ratio of the component along the (ox) axis to the axial component of the magnetic induction for a circular arc coil with a height of 120 mm, a radius of 100 mm, and three pole pairs, carrying a current of 3 A.

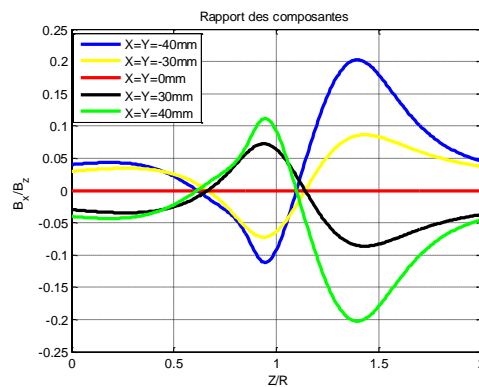


Fig. 15. Ratio of the following component (ox) to the following component (oz).

Displayed here are the curves of the ratio of the component along the (oy) axis to the component along the (oz) axis of the magnetic induction for a circular arc coil with a height of 120 mm, a radius of 100 mm, and three pole pairs, carrying a current of 3 A.

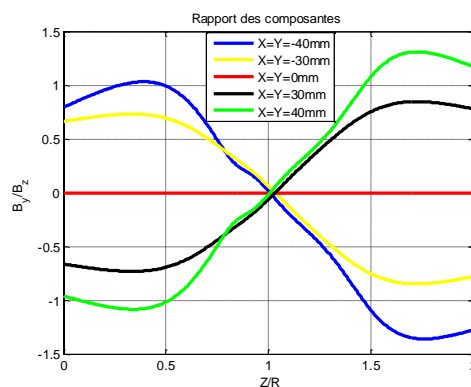


Fig. 16. Ratio of the following component (oy) to the following component (oz).

III.5 Components magnitudes

The magnitude of the magnetic induction is calculated using the following equation:

$$\|\vec{B}\| = \left(B_x^2 + B_y^2 + B_z^2 \right)^{\frac{1}{2}} \tag{53}$$

Displayed here are the curves of the magnitudes of the bars, arcs, and the system along the (oz) axis for a circular arc coil with a height of 120 mm, a radius of 100 mm, and three pole pairs, carrying a current of 3 A.

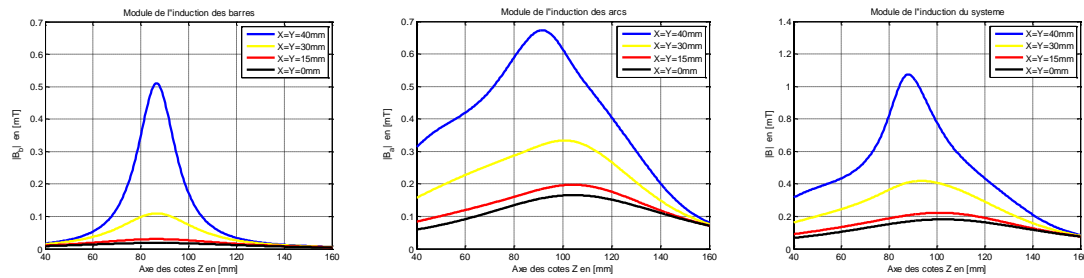


Fig. 17. Components modules.

IV. CONCLUSION

The main objective of this article is to contribute to the determination of the spatial components of the magnetic induction created at any point by a circular arc-like coil in a vacuum, representing the windings of rotating machines, using the Biot-Savart law. The complex shape of the coil necessitates the use of a Cartesian coordinate system, combining a concave circle and a rectangle. The primary challenge of this work is that the components have not been determined analytically due to the difficulties in calculating the integrals. For those interested in analytical results, it is preferable to use integral approximation methods (Simpson, trapezoidal, etc.) or to approximate the function (Lagrange approximation) and then integrate it, a process easily handled with MAPLE. The curves presented in this article are all plotted using MATLAB. The results indicate that it is possible to express the components of magnetic induction at any point in space. The graphical results revealed extremums, whose locations have not been studied.

The objective has been achieved, but this work has raised other issues, such as determining additional losses in electrical machines. These findings confirm the relevance of this article to the study of the electromagnetic compatibility of a device during its design.

REFERENCES

- [1]. G. Chateigner, D. Bouix, M. Boies, J. Vaillant et D. Verkindère, « Manuel de Génie électrique », Paris, Dunod, 2007, (ISBN 978 2 10 0484499 7)
- [2]. Lasne, « Exercice et problème d'électrotechnique », Paris, Dunod, 2005, (ISBN 2 10 049064 8)
- [3]. J. Laroche, « Introduction à l'électrotechnique – fondements d'électricité et électromagnétisme », Science Sup, Dunod, 2002.
- [4]. L. Lasne, « Electrotechnique pour la distribution d'énergie », Université de Bordeaux 1, 2004.
- [5]. G. Segulier et F. Notelet, « électrotechnique industrielle », deuxième édition, Lavoisier TEC et DOC, 1996.
- [6]. T. ALY SAANDY, « Influence de la géométrie des machines synchrones sur leur comportement face aux harmoniques de courant », Thèse de Doctorat de l'Université PARIS 6, Ecole supérieure d'électricité, 1994.
- [7]. S.R.Holm, H. Polinder, J. A. Ferreira, M. J. Hoeijmakers, P. van Gelder et R. Dill, « calcul analytique du champ magnétique dans des machines électriques dues à la densité de courant dans un enroulement de hauteur de fuite ». Dans la 15ème conférence internationale sur les machines électriques (ICEM 2002), Bruges, Belgique, 25-28 août 2002
- [8]. M. Marty, D. Dixneuf et D. G. Gilabert, « principe d'électrotechnique », Paris, Dunod, 2005, (ISBN 2 10 048550 4).
- [9]. Jeannot VELONTSOA, Tsialefity ALY SAANDY, Avisel Fredo TORO, Ulrich Michaël MAHAVELONA, Jean RALISON et Abdallah ATTOMANI. «Détermination analytique des composantes spatiales de l'induction magnétique créée par une spire circulaire ou rectangulaire ainsi que par un solénoïde en arc circulaire ou parallélepédique». Afrique Science, Vol.11, N°4 (2015), 1 juillet 2015. <http://www.afriquescience.info/document.php?id=4888>. ISSN 1813-548X.