American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-13, Issue-4, pp-96-109
www.ajer.org
Research Paper Open Access

A Global Strategy to Fault Diagnostics Using Multiaxial Fatigue Criterion: Application to Railway Track Holding Components

Tse Sparthan[1], Waindim Yisa[2], Wolfgang Nzie[3], and Tibi Beda[4]
[1] Department of Agricultural and Environmental Engineering, University of Bamenda, Bamenda, Cameroon
[2] Department of Mechanical Engineering, University of Bamenda, Bamenda, Cameroon
[3] Department of Mechanical Engineering, University of Ngaoundere, Ngaoundere, Cameroon
[4] Department of Physics, University of Ngaoundere, Ngaoundere, Cameroon
Corresponding Author: Tse Sparthan (sparthanazog@gmail.com)

ABSTRACT: *The huge traffic demand due to economic constrains from central African nations has increased the downtime of rolling stock infrastructure in Cameroon. Either due to derailment or other related failures. Damage due to fatigue account for about 90% of the entired failures and accidents in varieties of industries worldwide. Endure either from overloaded trains, locomotive vibrative effects, freight wagons movements, corrugation, heat, humidity and other environmental recesses in the rolling stock industry, a hand full of researchers have developed theories and experimental strategies to educate and improve the fatigue life of engineering components without looking into the aspect of fault diagnostic. This research paper focuses on suggesting with application a fault diagnostic strategy suitable to out practice failure discovery, extraction and segregation using Fogue multiaxial fatigue criterion and the Strength Analysis Technique in setting up a fatigue failure index. The multiaxial fatigue nature of the track components was assessed in other to establish a diagnostic index capable of qualitatively quantify the rate of fatigue failure when operating the rolling stock transportation sector, and by improving the maintenance process of railway track holding components in recent future era.*

KEYWORDS *Multiaxial Fatigue, Fogue Criterion, Failure Diagnostic, Infrastructure, Railway-Track.*

Date of Submission: 05-04-2024 Date of acceptance: 17-04-2024

I. INTRODUCTION

The livable of railway transportation industry strongly relies on the honesty and attainability of its holding systems. Their recesses during operation affects the arriving duration of passengers, freight, and also account for train related accidents caused by human mistakes, excessive train speed, deficient components, derailment and poor observation platforms. According to U.S department of transportation statistic report, an average of 75.4% train accidents is allied to derailment, 11.0% to collusion and 13.6% to other failures. 85% of mechanical recesses are reported from damage rail, bearing, axel, wheel, track geometry, defective wheel-rail contact surface, suspension springs impairment and wear of brake pads [1]. These recesses are the prime factors link to train derailment [2]. Endured either from overloaded trains, locomotive vibrative effects, freight wagons movements, corrugation, heat, humidity and other environmental recesses. Freights overloading increases the nature of stresses within components and weaken their material ingredient during usage [3]. These weaknesses inclusively known as fatigue is associated to about 90% of mechanical recesses related to downtime in the rolling stock industry [4], [5], [6], [7]. Animating material selection during design and manufacturing processes, recesses cause by fatigue is directly related to the creation, distribution and deep penetration of cracks gain from a slowly duplicated loading attitudes. In the rolling stock industry, fatigue damage will directly showcase its ability through the rate of material removal, stress changeability, corrosion, macroscopic cracks, deformation and complete rupture. To understand the fatigue attitudes of train-track holding components, reviewers distinguished from a laboratory viewpoint to model oriented strategies. The laboratory test favors prototype spacemen's and huge assumptions factors adopted on large components relating to either axial, rotational, bending, twisting and rolling contact fatigue analysis loading techniques. The fracture mechanic strategy globaly focuses on a pre-cracked spacemen put through a fixed-loading-amplitude nature. Being an analytical approach, the generated output signals gain from numerical analysis grant the posibility to quantify the depth of a crack vases the exact cycle counting limits in function of the stress intensity factor required to cause full rupture.

Relating to the dynamic nature of the rolling stock industry, huge demand is requested from scholars and engineers to cross examine and counteracts recesses related to fatigue damage over time. To mitigate on these issues, an approach that explores the bending and axial testing techniques in quantifying the fatigue threshold damage values during designing a high velocity rails, wheels, bogie frames and axels has been appraised with contrast to EN 13104 and ISO 12107 standards without field work [8], [9], [10]. To boost up the former, a wireless sensor that has the ability to register signals relating to stresses capable of enforcing fatigue damage on a rail under fixed and movable cycle counting loading behavior has been executed without experimental validation [11]. The use of finite element strategy to model and quantify the exact fatigue threshold values for cycle counting stresses and the remaining service life for both a heavy velocity bogie frames and the rail under thermal, traction and revolute contact nature was previously accepted with numerical and field solutions. The authors acknowledge that both the bogie and rail are under multiaxial loading nature during field evaluation which calls for future investigations [12], [13]. The use of finite element analysis to numerically investigate the nature of stresses on a subgrade when subjected to wheel/rail irregularities was further introduced with acceptance as the stresses on the top surface portrays a 70% damage increment contrast to that at the bottom [14]. Guilherme and Goncalo, combine the finite element analysis strategy and Palmgren-Miner's damage theory to estimate the exact threshold cycle counting stresses capable to startup a crack. Moreover, the authors were able to estimate the exact fatigue damage life spand required to maintain a good operating mode of a concrete bridge under thermal effects gain from heavy velocity trains without field accountability [15]. An EN13749 regulatory strength testing suitable to quantify the fatigue threshold for a heavy velocity train housing along a straight and curve railroad lines has also been inaugurated [16]. A strategy that combination Paris damage law and the Walkers equation to explain how three changeable loading natures influence the beginning, evolution and geometrical attitude of crack on the rail was further validated both theoretical and experimentally. However, the rate of deterioration between low cycle and high cycle fatigue was distinguished under uniaxial nature of stresses but multiaxial nature of assessment is still of huge interest. Ruilin and Sakdirat uses the bending moment technique to setup an improved maintenance strategy for mordern railway concrete sleepers in Australia. Using signals gain from field studies under static and vibrating nature caused by wheel/rail irregularities to established the crack growth attitude. The concept of distinguishing between uniaxial from multiaxial fatigue still calls for concern [17]. Sołkowski et *al*., investigate the level of fatigue damage on a polyurethane railroad mats using the servo-hydraulic compression testing rig to generate stress-strain signals suitable to estimate mechanically the bedding modulus and loss factor of two spacemen of slide material differencies. Their output favors the crack propagation strategy as it provides a huge ability to quantify heavy crack trajectory and durability of components in real time [18].

Inclusively, a huge amount of researchers has valoralised the aspect of fatigue behaviors of engineering material both theoretical and experimentally, but the application of multiaxial fatigue is limited with field appreciation. Also, the aspect of putting to service a diagnostic strategy center on fatigue criterions still required acknowledgement. This paper shall focus on suggesting an approach that is going to assist railway operators to improve on the maintenance quality of engineering systems subjected to mechanical fatigue failures using multiaxial fatigue criterion. For consistency, a cross examination on multiaxial fatigue standpoints and the elements of Fogue criterion shall be executed in section two. The suggested methodology to aid fault diagnostic with application to track holding components in section three and four. Meanwhile, conclusion and future research openings are detailed in five.

II. MULTIAXIAL FATIGUE THEORY

In reality, uniaxial fatigue criterions considered the nature of stresses on component in a single line of action. Their endurance limit and the fatigue damage threshold parameters in relation to the number of cycle for a given material is obtain under a unique line of execution. This assumption does not favor rolling stock holding components during field evaluation. Since they are subjected to tension, compression, twisting and/or hybrid attitudes. Multiaxial criterions iterate that the endurance limit and/or the fatigue damage threshold parameters in relation to the number of cycle for a given material is obtain when subjected to multiple axis of execution. This assumption favors the dynamic effect of multiple loading nature of the train on the track. To multiaxially access the endurance threshold required to enforce damage on a physical components, the selected criterion for fatigue analysis must intergrates information that expresses the constrains [$\sigma_{ij}(t)$] in relation to time changeability when passes through traction, deflection and torsion either symmetrical or repetitively [19]. Equation 1 expresses a unit quantity in fuctions of the various constraints on a component in three dimension. If damage function $E<1$ then the fatigue life of the given component is high as compare to the actual number of cycle (N); when $E=1$ the maximum fatigue life of the given component is equal the number of N cycles; and lastly, when $E>1$ the fatigue life of the given component is lower as contrast to the actual cycle (N). To wind up, when the function that quantify the fatigue damage index is a unit value, the service life of a given train-track holding component put through multiaxial loading stresses is directly identical to the useful life that expresses the endurance or fatigue damage index E.

E (Loading Constrains) = 1………. (1)

Plenty of scholars has cross examining the attitude of multiaxial fatigue nature on components subdue to complex variable loading problems in real time using quality benchmark. Separating from Empirical and the tempering of Coffin-Manson stand point to the stress/strain invariant, space average stress/strain and the energy concentration theory standpoints [19]. And also distinguished from Low cycle $[\mathrm{N} < 20000]$ to high cycle $[\mathrm{N} > 20000]$ multiaxial fatigue theories, these criterions focuses on problems that required mostly field work, a specific type of material, shape, size, the combination of stress/strain energy parameters to estimate the tiredness of a components subdue to recurrent loading situations. The critical plane standpoints center its activities on multiaxial fatigue problems whose preciseness is based on the search for a critical plane along the normal or tengential when put through complex loading [20]. This stand point favors the multiaxial fatigue assessment of rolling stock holding components as their loading nature during field evaluation is random and requires a sever approach to fully estimate the directions at which the stresses are oriented. Secondly, to determine and quantify the line of action at which a recess can be quickly encounter on the given components. This approach favors the Fogue multiaxial fatigue viewpoint. Fogue suggested a cross exanimating criterion which grant the possibilities for engineers to be able to qualitatively quantify the high cycle multiaxial fatigue damage of industrial components under recurrent loading. The criterion differs from others as its gives the posibility to cross examine the fatigue life of industrial components using the amplitude (σ_{ha}) and mean value of the shear strain (σ_{hm}), the amplitude of the average shear stresses (τ_{ha}) as their key performance indicators to track the failure patterns with respect to time (equation 2). σ_{hmin}, σ_{hmax} and $\Delta\sigma_{ha}$are minimum, maximum and range of shear strain respective fatigue parameters define alone the normal plane h and origin p; a, b and k are the Haigh constants obtain during field calibration. Torsion has a slope identical to the model obtained in traction and compression; τ_{hm} is the mean value of the shear stress acting directly on component plane and globaly potray no object of interest during real time evaluation of fatigue failure $(c = 0)$[19], [21].

$$E_{FG} = \left[\int_{s} s^{-1} * \left(\frac{a\tau_{ha}+b\sigma_{ha}+c|\tau_{hm}|+d\sigma_{hm}}{\sigma_{-1}}\right)^{2} d\mathbf{s}\right]^{0.5} \ldots\ldots\ \mathbf{(2)}$$

$$\sigma_{ha} = \frac{\sigma_{hmax}-\sigma_{hmin}}{2}\ ;\ \sigma_{hm} = \frac{\sigma_{hmax}+\sigma_{hmin}}{2}\ ;\ \sigma_{ha}(t) = \sigma_{h}(t) - \sigma_{ha}(t)$$

$$\Delta\sigma_{h} = \sigma_{hmax} - \sigma_{hmin}\ ;\ \sigma_{h} = \Delta\sigma_{h} * sin\left(\frac{2\pi}{T} * t\right)\ ;\ \tau_{ha}(t) = \tau_{h}(t) - \tau_{hm}(t)$$

$$a = \sqrt{0.5 * \left(12\left(\frac{\sigma_{-1}}{\tau_{-1}}\right)^{2} + \left[\left\{0.5 * \left(15 - \left[9\left(25 - 8\left[\left(\frac{\sigma_{-1}}{\tau_{-1}}\right)^{2} - 3\right]^{2}\right)\right]^{0.5}\right)\right\}^{0.5}\right]^{2}\right)}$$

$$b = \sqrt{0.5 * \left(15 - \left[9\left(25 - 8\left[\left(\frac{\sigma_{-1}}{\tau_{-1}}\right)^{2} - 3\right]^{2}\right)\right]^{0.5}\right)}$$

$$d = \frac{-(2a+3b)}{3} + \frac{\left[(2a+3\ b)^{2}+45\left(4\left(\frac{\sigma_{-1}}{\tau_{-1}}\right)^{2}-1\right)\right]^{0.5}}{3}$$

III. SUGGESTED DIAGNOSTIC STRATEGY

In advance engineering, the developing state of any mechanical system relies strictly on various physics concepts that analytically cross examine their static and dynamic attitudes when subdue to internal or external loading effects. Therefore, to effectively cross examine the various healthy aspects of the entired rolling stock systems put through mechanical misfortune caused by fatigue, researchers and field operators in recent are call to develop active and proactive analytical strategies to aid failure discovery, segregation and components useful life quantification in real time. Though, plenty has insisted on suggesting prognostic strategies that focuses on the use of past signals gain from field work using material fatigue laws to establised mechanical failures deteriorating patterns, none has suggested a unique strategy which is suitable to universally perform failure detection, extraction and segregation on engineering parts with fatigue as the object of interest. The complained about inconsistency of field collected signals and the cost escalation of prototype installations that relaxes the quality of the signal oriented strategies and favors analytical strategies centered on physic laws. Meaning, at the developing stage of any engineering component, if the healthy aspects can be established with analytical models center on fatigue damage threshold level, then fig. 1 is a suggested strategy that should be exploited by rolling stock design and maintenance engineers to appraised and improved on the sustainability of track holding systems in real time.

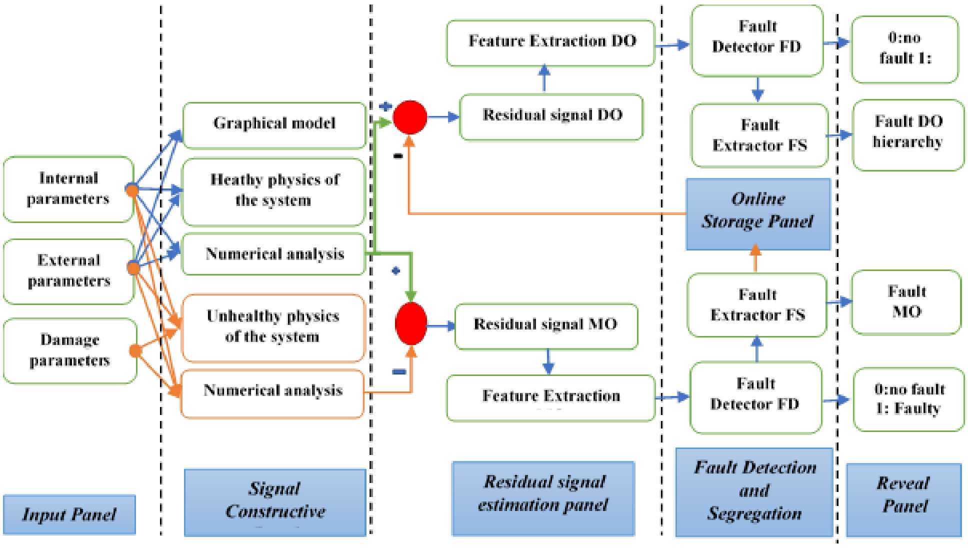

Fig. 1 The adopted methodology for damage detection and segregation

The input panel outline qualitative signals of the system to be investigated with strict consideration on the flexibility nature of mechanical parts, internal and external environmental data's that influence its smooth operating conditions. Thus, stipulate a multiaxial recurrent stress-strain state series and put to service a unique pattern that serves as the multiaxial cycle counting identity, consider a unique Fogue multiaxial fatigue damage criterion that favors the failure patern influenced either by external/internal stresses, and lastly identify the service duration of every component (equation. 3) for the track holding components. Table 1 is an extract that showcases the characteristic properties adopted for estimating the required number of cycles needed with respect to their individual design periods (years). T_D : allowable working duration for the vehicle/track component; T_P: number of trips per day for a unique train; T_{PN}: number of train passes per day; T_W: number of a unique train travel per week; T_N : number of weeks a unique train travel per year; R_L : length of the railroad(KM); T_S: Train speed. The number of estimated cycles required for the train holding components defers from that of the track because they are design to be used in a short duration as contrast to the track.

$$\text{Track}: (N^T)_i = (3600 * T_D * T_{PN} * T_W * T_{NW})/(T_N * T_S) \qquad (3)$$

Table 1. Properties adopted for estimating the required number of cycles $(N^T)_i$

Ref.	Parameters	Train Components	Track Components	SI Units
1	TP	2	2	
2	TD	12	15	Years
3	TPN		8	
4	TW	4	4	
5	TNW	52	52	
6	RL	263		
7	TS	40	40	Km/h
8	TN	0.1	0.01	Seconds

The signal constructive panel begins a model that incorporate the stress-strain behavior of the components. Next, an updated analytical model that welcomes variability of the entired system within a negative context is further simulated to establised poor signals that showcases multiaxial high cycle fatigue level of every component. From kelvin Voight rheological principle, the mechanism of failure for the track holding components relies on the amount within which each components undergoes distortion before failure. The modelling of fatigue damage to aid failure discovery, segregation and speculation of the track holding systems in the multiaxial loading scenarios relies on equation 1 as potray in [21]. Since the Fogue criterion can easily be given a unit quantity in fuctions of the various constraints and an endurance limits or the limit of cycle counting simutanously (equation 4 and equation 5). Meaning, damage discovery and quantification of various rolling stock holding components put through fluctuating loading (F^T) can be estimated when the function that associates the fatigue threshold is in line with the pattern $\langle (E_{FG}^T)_{N^T} \in [0\ 1] \rangle$. As (E_h^T) and expresses the crack evolution rate in the Fogue damage index. This can be to set-up a diagnostic and/or prognostic strategy on rolling stock components.

$$(E_h^T) = \frac{a(\tau_{ha})^T + b(\sigma_{hha})^T + d(\sigma_{hhm})^T}{(\sigma_{-1})^T} \quad (4)$$

$$(E_{FG}^T)^2 = \left[s^{-1} * \int_S (E_h^T)^2 ds\right] \quad (5)$$

$$\text{acceptable } interval: \begin{cases} \frac{1}{\sqrt{3}} \leq \frac{{\tau_{-1}}^T}{{\sigma_{-1}}^T} \leq \frac{\sqrt{3}}{2} \\ \frac{1}{2} < \frac{{\sigma_{-1}}^T}{{\sigma_0}^T} < 1 \end{cases}$$

Where: ${(\tau_{ha})^T}_i$, . ${(\sigma_{hha})^T}_i$, ${(\sigma_{hhm})^T}_i$are the amplitude of the alternating tengential shear, normal stresses and the mean value of the normal stresses required to express the fatigue parameters for a unique holding components define along the plane h^T and origin p during the cycle (N^T). τ_{hm} is the mean value of the alternating tangential stresses, $\tau_{hm} = 0$ due to the fact that its portrays very less influences regarding the material damage put through multiaxial fatigue stresses. Moreover, to perform fault diagnostic using the Fogue criterion, the health free state is gain when have equation 5 transform to equation 6; unhealthy state when we have equation 7. $(F_{cr}^T)_i$ is the critical value of the train loading parameter correlating to a critical number of cycle $({N^T}_{cr})$ associating to a critical crack level $(E_h^T)_{cr}$ that triggers a change which may lead to sudden or slow damage. F_{max}^T and F_{min}^T are maximum and minimum loading quantities that the train holding components are put through during static and dynamic full usage. To established the fault diagnoser, the signal patterns that showcases the healthy and unhealthy state of the track holding components relies on a numerical resolutions of the modified equations (2-7) with strict consideration of system data as we all know that the quality of a good failure discovery and segregation tool hold esteem advantages on its input signatures

$$Healthy\ state\ is\ gain\ when: \begin{cases} \left((E_{FG}^T)_{N^T}\right)_i < 1 \\ \\ (F_{min}^T)_i \leq (F_{cr}^T)_i < (F_{max}^T)_i \end{cases} \quad (6)$$

$$Un\ Healthy\ state\ is\ gain\ when: \begin{cases} \left((E_{FG}^T)_{N^T}\right)_i \geq 1 \\ \\ (F_{cr}^T)_i \geq (F_{max}^T)_i \end{cases} \quad (7)$$

Residual signal is a friendly word used in the arena of condition-based maintenance plus applicable to industrial systems. It grants the posibility to quantify qualitative features required to perform failure discovery and segregation on linear or non-linear systems. As diagnostic is a static segregation of healthy from unhealthy state of affair with respect to time in the rolling stock industries. Putting to service first rated features that has the ability to watch, appraise and estimate the robustness of the train-track holding components is of huge relevant to guarantee the safety of goods and services. As its thus, assist in excerpting hidden information found in natural signals using monotonibility and trendability way of doing [22]. The choice of technique for feature first rating relies on the failure pattern of the given component. Some practical applications may call for a hybrid technique depending on the decrement/increment of the natural signal [23], [24]. Practical evaluation of mechanical failures for most rolling stock parts under variable loading globally relies on the number of cycle counting establised to quantify low and high cycle fatigue stresses. These paradoxes make life favorable to manage fault discovery and segregation based on pattern of failure. The interactive nature within the track holding components during field work count on high esteem signals to established their health state of affairs. The quality of the excerpt and first rated features is the aspiration of attentiveness. Features are excerpted in stages, residual signals are gained based on variation within threshold signals and the signals obtained as the system deteriorates over time (Fig. 2). Equation 8 is used to creates excerpted features based on the residual equations (6-7) gain from multiaxial stress values that portrays a reduction for every $({N^T}_{cr})_k$ under irregular loading phenomena with $(\%R_{FG})_i^q$ the residual signal calculated.

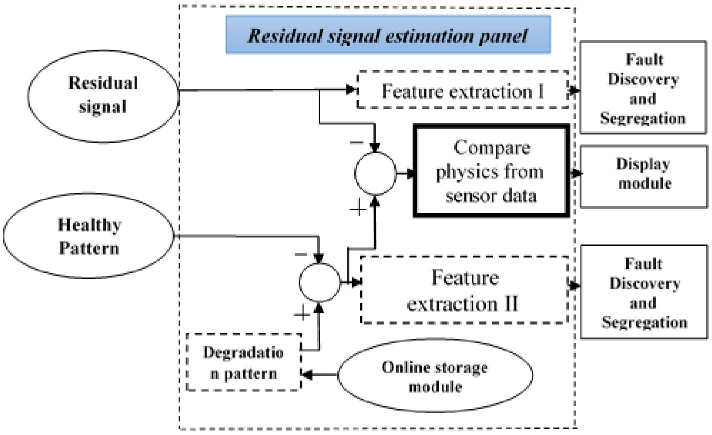

Fig. 2. Internal view of the residual signal estimated Panel [23], [24]

$$(\%R_{FG})_i^q = \left[\frac{(\Delta E_{FG}^T)_{FT_{cr}} - (\Delta E_{FG}^T)_{FT_{cr+1}}}{(\Delta E_{FG}^T)_{FT_{cr}}}\right]_i x\ 100 \qquad \textbf{(8)}$$

The fault detection and segregation panels extract and classify the different unhealthy state of the entired system from its comfort zone. The faulty situation is centered on features extracted from the residual patterns developed. Regarding the non-linearity of track systems, the direct variability of the feature patterns gain from individual components are used to perform failure discovery and isolation actions within these section. This concept grants the posibility to directly understand and manage non linearity issues, perform discovery and extraction issues using less computational duration with applications. Meanwhile, the reveal panel selves as a screen were all the output signals of the different healthy and unhealthy scenarios with their classifications are visualized through an LCD or Computer Interface to be constructed. These signals are further stored online using a computer unit through wireless connections or offline using an external memory card for future appraisal of the above diagnostic strategy using historical techniques.

IV. Application to Train-Track Holding Components

Input Panel: The quick discovery of failures in the rolling stock industries relies strictly on the legit extraction of components parameters required to outperform fault diagnostic in real time. However, the attitude of failure in track components differs, but their mechanics of failure pattern due to stresses gain from both internal and external excitations are uniform. The suggested strategy is valuable as it focused on the given component and it failure parameter at a time. Statistically, the engineering misfortunes that destabilizes the esteem usage of track holding parts is globaly associates to 90% of fatigue failure caused by recurrent loading gain from train traction forces either during static/and or dynamics scenarios, and variation on uncomfortable loading cycles either uniaxial or multiaxially. Considering fatigue threshold as the key performance indicator to aid diagnostic, the external and internal ingredients of track holding components to be case study in this piece of research are in (table 1 & 2), and also the failure ingredient centers on multiaxial stresses (E_{FG}^T (N^T)). Fig. 1 is the full suggested model that uses multiaxial fatigue stress criterion suggested in this paper to calculate failure of the track holding components. Feature extraction signals are segregated from model $\left[(\%\ \boldsymbol{R_{FG}^T})_{\boldsymbol{i}}\right]^{\mathbf{1}}$ to data $\left[(\%\ \boldsymbol{R_{FG}^T})_{\boldsymbol{i}}\right]^{\mathbf{2}}$ and analytical/signals $\left[(\%\ \boldsymbol{R_{FG}^T})_{\boldsymbol{i}}\right]^{\mathbf{3}}$ oriented for strategy validation and residual signals quality control. To estimate the level of fatigue damage in a component put through dynamic loading scenarios ($F_{cr}^{\boldsymbol{T}}$), the crack initiation parameter $(E_h^T)_i$ that grant the ability to estimate the fatigue damage function $(E_{FG}^T)_i$ is of prime interest. For the sake of this paper, the amplitude and mean stresses of various holding components are iterated with strict accordance to their various stress histories $\left[\sigma^T{}_{ij}(t)\right]$ and the fatigue test parameters $(\sigma_0^T\ ;\ \sigma_{-1}^T\ ; \tau_{-1}^T)_i$ in function of cycle counting gain from every loading increase with respect to time. The railway track key holding components are the rail and sleeper (table 2).

Table 2 Technical characteristics UIC railway Track [24].

System					
Track holding components					
EN13674-1 / 54E2 1084 medium carbon steel rail			*Hot rolled structural EN10025-S275-600 grade Sleeper*		
Designation	***Values***	***SI Units***	***Designation***	***Values***	***SI Units***
Elongation ; $\boldsymbol{\varepsilon^R}$	10	10%	Plate weight S_w	39.53	kg/m
Inertia moment: $\boldsymbol{I^R}$	2127	cm^4	Section high S_A	115	mm
Cross section: $\boldsymbol{A^R}$	$69.34 * 10^{-4}$	m^2	Section width S_B	280	mm
Density: $\boldsymbol{\rho^R}$	7850	kg/m^3	Rail set width: S_C	168	mm
Mass per meter:	54.77	kg/m	Rail seat thickness: S_D	14.5	mm
Poison ratio $\boldsymbol{\vartheta^R}$	0.28		Leg thickness: S_E	7.6	mm
High of neutral axis: $\boldsymbol{X^R}$	791	mm	Moment of inertia: S_{XX}	654.7	cm^4
Yield strength: $\boldsymbol{\sigma_y^R}$	668.8	MPa	Section modulus: S_V	81.3	cm^3
Ultimate Tensile Strength: $\boldsymbol{\sigma_U^R}$	930.8	MPa	High of neutral axis:S_F	80.5	mm
Young modulus: $\boldsymbol{E^R}$	210	GPa	Ultimate tensile strength: σ_u^{sp}	590	MPa
Service life: $\boldsymbol{N_R}$	$1.12 * 10^{11}$	$Cycles$	Yield strength: σ_y^{sp}	410	MPa
Shear strength: $\boldsymbol{\sigma_S^R}$	551.6	MPa	Elongation: ε^R	≥ 23	%
Gage: $\boldsymbol{G^R}$	1.435	m	Cross section: A_{sp}	$54.9 * 10^{-3}$	m^2
Height: $\boldsymbol{H^R}$	0.161	m	Service life N_{sp}	$1.12 * 10^{11}$	$Cycles$
Head width: $\boldsymbol{h^R}$	0.067	m	Sleepers length :L^{SP}	2500	mm
Web thickness: $\boldsymbol{w^R}$	16	m	Maximum load per spring: F_{max}^T	298.92	kN
Foot width:$\boldsymbol{f^R}$	0.125	m	Carbon	≤ 0.21	
Maximum load per spring:$\boldsymbol{F_{max}^T}$	298.92	kN	Manganese	≤ 1.50	
Equivalent Profile	UIC54	$-$	Phosphorus	≤ 0.035	
Carbon	≤ 0.30		Sulfur	≤ 0.035	
Manganese	$0.7 - 1.00$		Silicon	0.14-0.25	

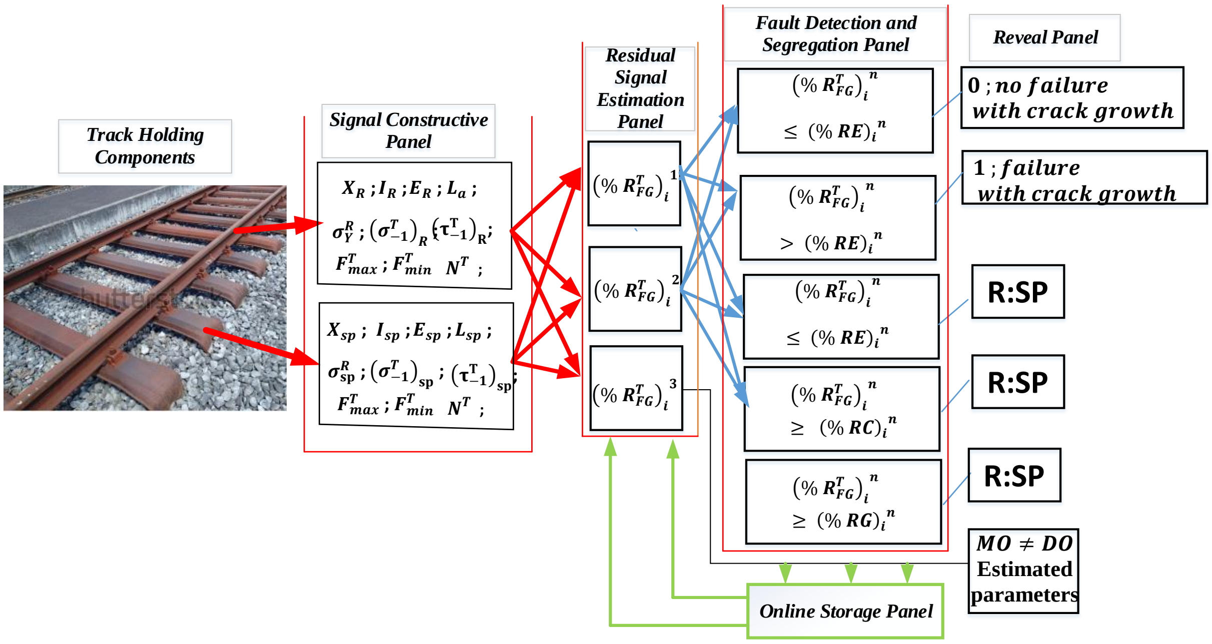

Fig. 3. Orientation of suggested model for fatigue failure discovery and segregation

Signal Constructive Panel: Conceived to guided and protect the livelihood of rolling stock vehicles either fixed or on motion, the track components showcase huge material damage due to mechanical failures triggered by heavy stresses that leads to material fatigue failure (Fig. 4). During real time exploitation, contact stresses are gain when the rail vehicle is put through accelerating or decelerating, thermal stresses are gain from temperature modification. Moreover, to assess the fatigue damage attitude of the track holding components when put through stresses, thermal stresses are regarded as preloaded stresses because of their inferiority over bending and contact stresses. The speed of the rail vehicle is uniform and as such, the contact stresses are negligible during multiaxial fatigue assessment. Therefore, equation 9 is an extract that grant the posibility to investigate the maximum bending stresses of the track components $(\sigma_{max}^{t})_{T}$ undser train vertical loading [25]; $(M^{T})_{st}$ is the track static bending moment, $E_T\,I_T$ is the flexural rigidity of the track holding components, C^t dimension between the base of the given components to its neutral axis, ρ^t rotational proportionality and K^t track stiffness, F^T train load and w^t relative deflection. The maximum bending moment of the rail when registered as a continues beam mounted on a rigid discrete supported layers put through train dynamic loading is gain from equations (10-12). l_{a} , α_{dyn} and V_t being the distance between two sleepers at mid span, the dynamic speed parameter and train speed particularly.

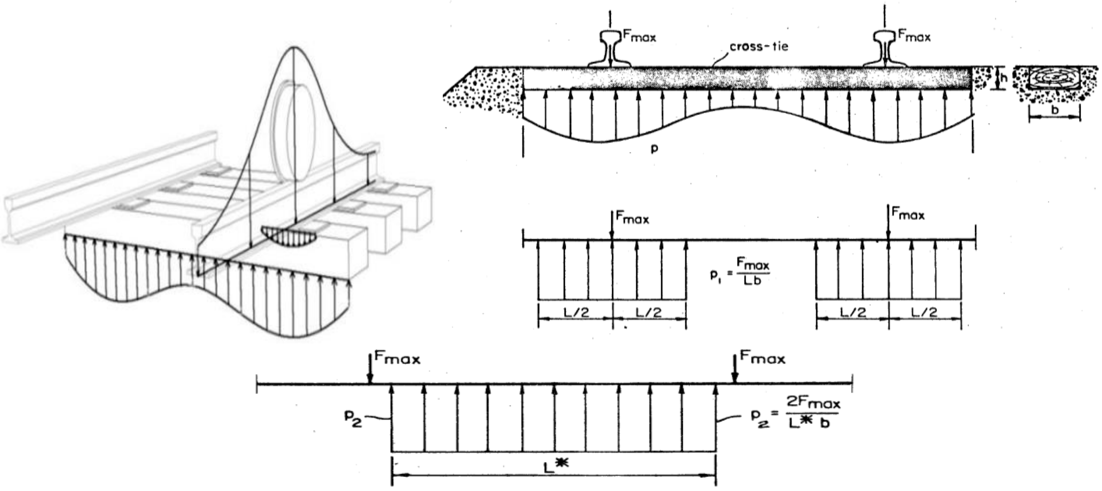

Fig. 4 Stress distribution in track holding components [25].

$$(\sigma_{max}^{t})_{T} = \left(\frac{E^t C^t M^T}{E\,I}\right)_T \quad (9)$$

$$\left(EI\,\frac{d^4w^t}{dx^4} - \rho^t\,\frac{d^2w^t}{dx^2} + K^t w^t = Q^t\right)_T \quad (10)$$

$$F^T\,(x) = K^t w^t(x)$$

$$(M^R)_{st} = \left(\frac{12\,l_{m}\,l_n - 7\,(l_{m}+l_n)+4}{16\,[3(l_{m}\,l_n)-(l_{m}+l_n)]}\quad F^T\,l_a\right)_{stat} \quad (11).$$

$$(M^R)_{dyn} = \alpha_{dyn}\left(\frac{12\,l_{m}\,l_n - 7\,(l_{m}+l_n)+4}{16\,[3(l_{m}\,l_n)-(l_{m}+l_n)]}\quad F^T\,l_a\right)_{stat} \quad (12)$$

$$\alpha_{dyn} = 1 + 4.5 * 10^{-5}(V_t)^2 - 1.5 * 10^{-5}(V_t)^3 \;:\; with\; V_t < 170\; km/h$$

$$l_{m} = 4l_{a}\,;\; l_{n} = 3l_{a}\,;$$

However, the maximum bending stresses of the sleepers is gain when the bending moment is at its maximum $\left[(M^S)_{dyn} = \alpha_{dyn}\,(M^{sp})_{st}\right]$ and respect the loading condition of fig. 4. L and L^* showcasing the width of the rail foot and the sleepers distance within two opposite rail foots on same route, b width of the sleepers. The stresses produced in sleepers are not uniform through its entired length. To estimate the maximum stresses, equation 13 will be used to quantify the sleepers stresses at rail foot $\left(\sigma_{rail\;foot}^{t}\right)_{sp}$ and at mid span $\left(\sigma_{mid\;span}^{t}\right)_{sp}$.

$$\begin{cases} \left(\sigma_{rail\;foot}^{t}\right)_{sp} = \left(\frac{F^T}{L\,b}\right)_{sp} \\ \left(\sigma_{mid\;span}^{t}\right)_{sp} = \left(\frac{2\,F^T}{L\,b}\right)_{sp} \end{cases} \quad (13)$$

To established quality signatures required to out practice failure discovery, extraction and segregation, numerical simulation is extrapolated base on the stress distribution attitude of the respective holding components when put through variable train loading scenarios in various categories as showcased (Fig. 5). The requested criterion and the orientation of the principal stresses are to be selected for every component.

Properties that depicts the fatigue threshold of various material for every component are deduced. Next, the crack evolution rate $\left((E_h^T)^2{}_{N^T}\right)_i$ is estimated and further used in quantifying the multiaxial fatigue threshold values $\left((E_{FG}^T)^2{}_{N^T}\right)_i$ of same components with respect to a 10% loading increase of the freights acting on the entired railway track.

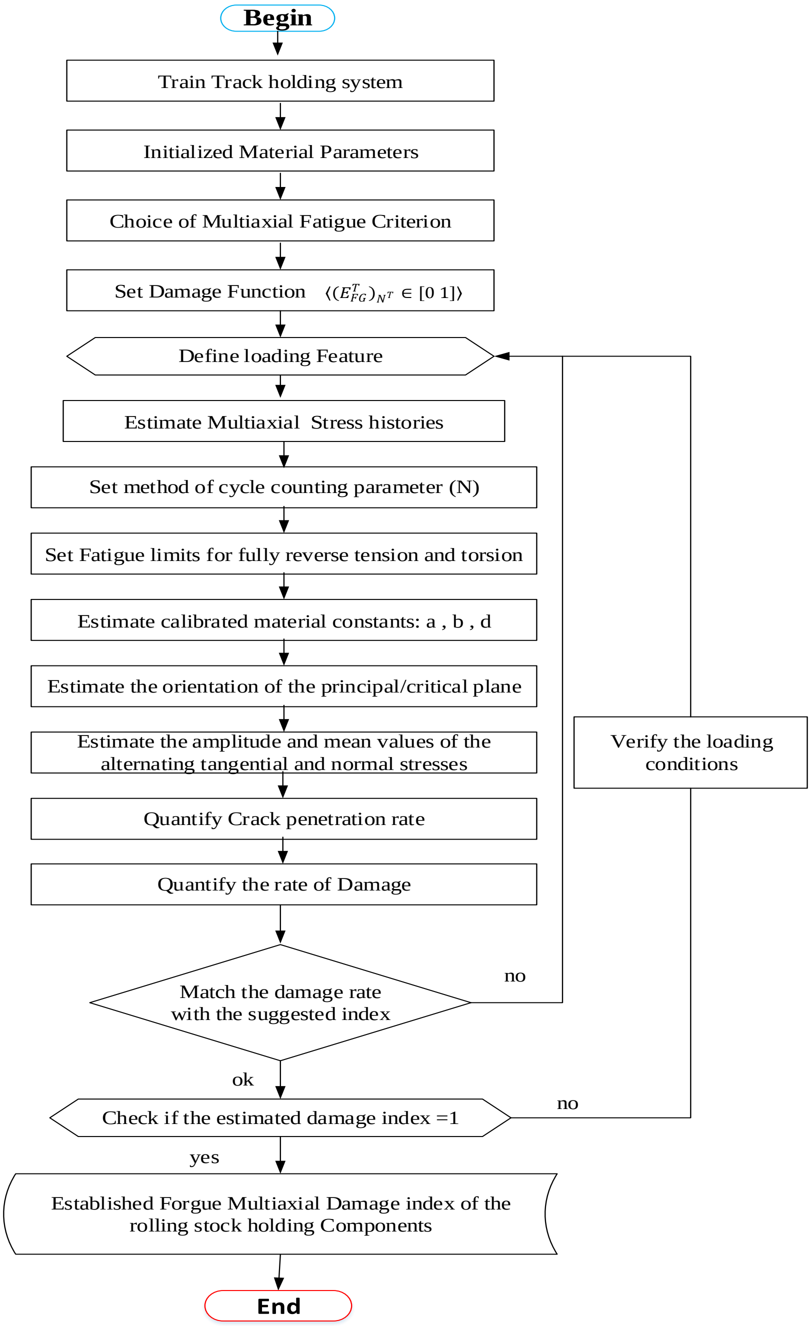

Fig. 5 The suggested Algorithm for failure discovery, extraction and segregation

To estimate the fatigue damage of the entired track key holding components, its preferable to establised the damage index for individual components. Table 3 showcases the estimated stresses for the rail with fatigue limits for completely reverses tension and torsion as $(\sigma_{-1}^T)_R = 361.94\ MPa$ and $(\tau_{-1}^T)_R = 208.84\ MPa$. When the freight nature (F^T) gain a critical value of $F_{cr}^T = 325\ kN$, the attitude of the shear stresses acting on the rail meet a minimum and maximum values of $\left(\boldsymbol{\tau}_{hhmin}^T\right)_R = 1462.31\ MPa$ and $\left(\boldsymbol{\tau}_{hhmax}^T\right)_R = 1746.58\ MPa$. Meanwhile the generalised estimated values of the amplitude and mean shear stresses that redirect the line of action of the critical plane that leads to crack initiation respects $\left(\tau_{hha}^{T}\right)_R = 142.14MPa$ and $\left(\tau_{hhm}^{T}\right)_R = 1604.44\ MPa$ orderly.

Table 3. Rail estimated under 10% Train loading increment (F^T) = F_{cr}^T

Samples	F^T (KN)	$\left(\tau_{hhmin}^T\right)_R$ (MPa)	$\left(\tau_{hhmax}^T\right)_R$ (MPa)	$\left(\tau_{hha}^T\right)_R$ (MPa)	$\left(\tau_{hhm}^T\right)_R$ (MPa)	$\left(\frac{\tau_{hha}^T}{\tau_{hhm}^T}\right)_R$
1.00	50.00	208.90	249.51	20.31	229.21	0.09
2.00	75.00	313.35	374.27	30.46	343.81	0.09
3.00	100.00	417.80	499.02	40.61	458.41	0.09
4.00	125.00	522.25	623.78	50.76	573.02	0.09
5.00	150.00	626.70	748.53	60.92	687.62	0.09
6.00	175.00	731.15	873.29	71.07	802.22	0.09
7.00	200.00	835.60	998.05	81.22	916.83	0.09
8.00	225.00	940.06	1122.80	91.37	1031.43	0.09
9.00	250.00	1044.51	1247.56	101.53	1146.03	0.09
10.00	275.00	1148.96	1372.31	111.68	1260.63	0.09
11.00	300.00	1253.41	1497.07	121.83	1375.24	0.09
12.00	325.00	1357.86	1621.83	131.98	1489.84	0.09
13.00	**350.00**	**1462.31**	**1746.58**	**142.14**	**1604.44**	**0.09**
14.00	**375.00**	**1566.76**	**1871.34**	**152.29**	**1719.05**	**0.09**
15.00	**400.00**	**1671.21**	**1996.09**	**162.44**	**1833.65**	**0.09**

Table 4 showcases the estimated stresses for sleeper with fatigue limits for completely reverses tension and torsion as $(\sigma_{-1}^T)_R = 361.94\ MPa$ and $(\tau_{-1}^T)_R = 208.84\ MPa$. When the freight nature (F^T) has a critical running value of $F_{cr}^T = 375\ kN$, the attitude of the stresses acting on the sleepers also gains its minimum and maximum values as $\left(\tau_{hhmin}^T\right)_{sp} = 624.99\ MPa$ and $\left(\tau_{hhmax}^T\right)_{ou} = 1237.77\ MPa$ successfully. The globalized estimated values of the amplitude and mean shear stresses that redirect the line of action of the critical plane at which the crack on the Sleepers are initiated respects $\left(\tau_{hha}^T\right)_{ou} = 306.39$ and $\left(\tau_{hhm}^T\right)_{ou} = 931.38\ MPa$.

Table 3. Rail estimated under 10% Train loading increment (F^T) = F_{cr}^T

Samples	F^T (KN)	$\left(\tau_{hhmin}^T\right)_R$ (MPa)	$\left(\tau_{hhmax}^T\right)_R$ (MPa)	$\left(\tau_{hha}^T\right)_R$ (MPa)	$\left(\tau_{hhm}^T\right)_R$ (MPa)	$\left(\frac{\tau_{hha}^T}{\tau_{hhm}^T}\right)_R$
1.00	50.00	83.33	165.04	40.85	124.18	0.33
2.00	75.00	125.00	247.55	61.28	186.28	0.33
3.00	100.00	166.66	330.07	81.70	248.37	0.33
4.00	125.00	208.33	412.59	102.13	310.46	0.33
5.00	150.00	250.00	495.11	122.56	372.55	0.33
6.00	175.00	291.66	577.63	142.98	434.64	0.33
7.00	200.00	333.33	660.14	163.41	496.74	0.33
8.00	225.00	374.99	742.66	183.83	558.83	0.33
9.00	250.00	416.66	825.18	204.26	620.92	0.33
10.00	275.00	458.33	907.70	224.69	683.01	0.33
11.00	300.00	499.99	990.22	245.11	745.10	0.33
12.00	325.00	541.66	1072.73	265.54	807.20	0.33
13.00	350.00	583.32	1155.25	285.96	869.29	0.33
14.00	**375.00**	**624.99**	**1237.77**	**306.39**	**931.38**	**0.33**
15.00	**400.00**	**666.66**	**1320.29**	**326.82**	**993.47**	**0.33**

Residual Signal Constructive Panel: The interactive nature within the track holding components during field work count on high esteem features to distinguish their health state of affairs. However, features shall be excerpted in stages since residual signals are gained based on variation within threshold signals and the signals obtained as the rail and sleeper deteriorates over time. Fig. 6 is an eyeshot's that expresses the nature of the rail damage index and the attitude of the features gain from residual signals when influence by train loading with huge increment. differentiating the feature gained from residual signals as Acceptable working state

$\left[(\%R_{FG})_R^q < 100\%\right]$ when $(E_{FG})_R^q < 1$ and the train load is within $F^T = F^T{}_{cr} < 325\ kN$, then the rail is in good health though it's may undergo some deterioration within $[0.15 - 0.93]$ but still stays within its elastic limit. When the features gain from the residual pattern welcome the state $\left[(\%R_{FG})_R^q = 100\%\right]$and also, the damage index and the entired train load respect $(E_{FG})_R^q = 1;\ F^T = F^T{}_{cr} = 325\ kN$, then the rail is at a critical operating point where damage due to fatigue may occur at any given moment since the rail deflection respects the plastic domain with constant deformation or immediate rupture. Circumstancies that defer the features gained from residual signals as Grevious working state $\left[(\%R_{FG})_R^q > 100\%\right]$ when $(E_{FG})_R^q > 1$ and the test value for the entired train load is within $F^T = F^T{}_{cr} > 325kN$, then the rail will register immediate rupture.

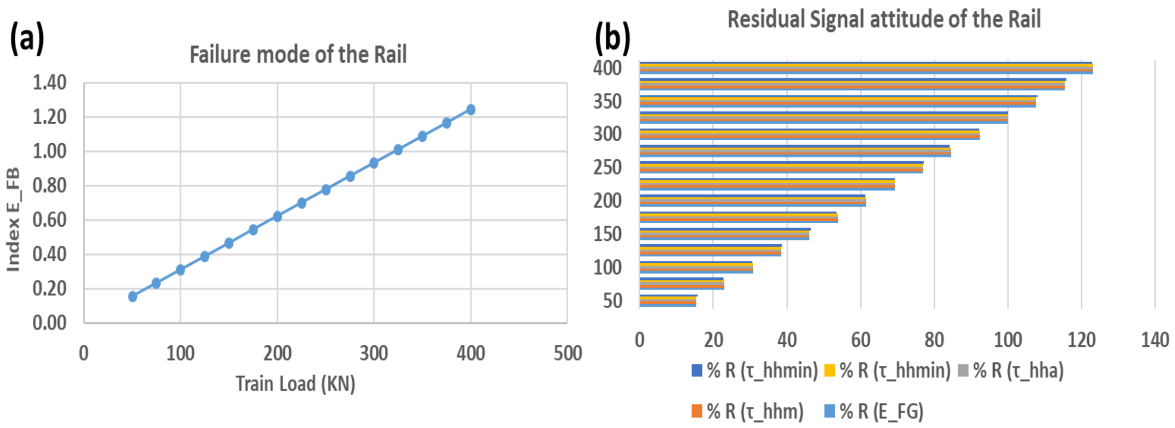

Fig .6 Rail (a) evolution of failure : (b) generated residual signals $(\%R_{FG})_R^q$

Fig. 7 is eyeshot's that expresses the nature of its damage index and the attitude of the features gain from residual signals for the sleepers. differentiating the feature gained from residual signals as Acceptable working state $\left[(\%R_{FG})_{sp}^q < 100\%\right]$ when $(E_{FG})_{sp}^q < 1$ and the test value for the train load is within $F^T = F^T{}_{cr} < 375\ kN$, then the sleepers is in good health though it's may have pass through a certain amount of fatigue effects within $[0.14 - 0.96]$ and within its elasticity. When the features gain from the residual pattern welcome the state $\left[(\%R_{FG})_{sp}^q = 100\%\right]$and also, the damage index and train load respect $(E_{FG})_{sp}^q = 1;\ F^T = F^T{}_{cr} = 375\ kN$, then the sleeper is at a critical operating point where damage may occur, the sleeper's deflection disrespects its elastic nature and respect the plastic domain with constant deformation or immediate rupture. Circumstancies that defers the features gained from residual signals as Grevious $\left[(\%R_{FG})_{sp}^q > 100\%\right]$, when $(E_{FG})_{sp}^q > 1$ and the test value for the entired train load is within $F^T = F^T{}_{cr} > 375kN$, then immediate rupture of the sleepers.

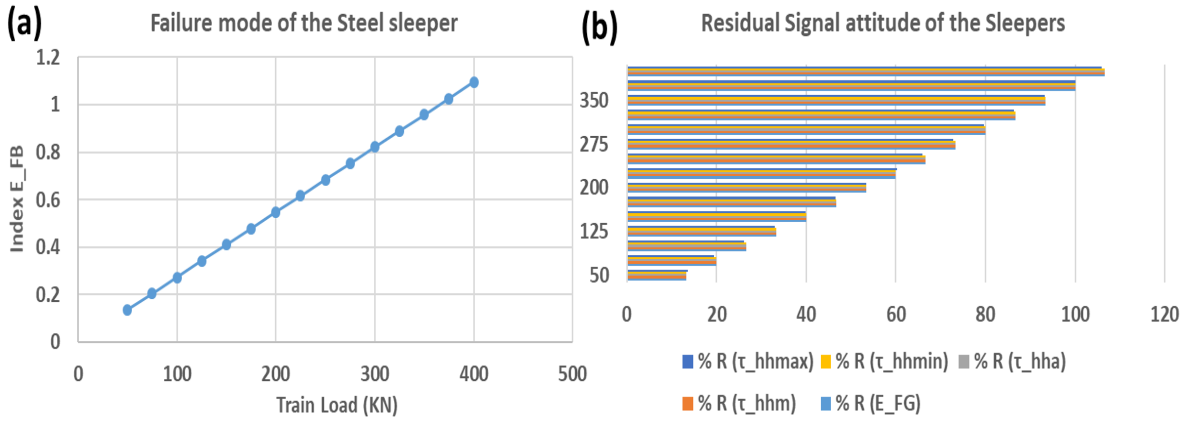

Fig .7 Sleeper (a) evolution of failure : (b) generated residual signals $(\%R_{FG})_R^q$

Fault Discovery and Segregation Panel: **Fig.7** is an eyeshot that expresses the contrast within the track holding components when put through variable loading nature gain from the rolling stock vehicle. Zero fault is registered when the feature gain from the residual signal respect $\left[(\%R_{FG})_i^q < 100\%\right]$, when the damage index is within $(E_{FG})_i^q < 1$ and the test value for the train load is within $F^T < {F^T}_{cr} = (325\,kN\,; 375kN)$ corresponding to the rail and sleepers accordingly. Failure is register immediately as the value of the corresponding residual signal respect $\left[(\%R_{FG})_i^q \geq 100\%\right]$ and the damage index for the rail and sleepers stays within $(E_{FB})_i^q \geq 1$ and the test value for the train load is within $F^T < {F^T}_{cr} \geq (325\,kN\,; 375kN)$ during field evaluation.

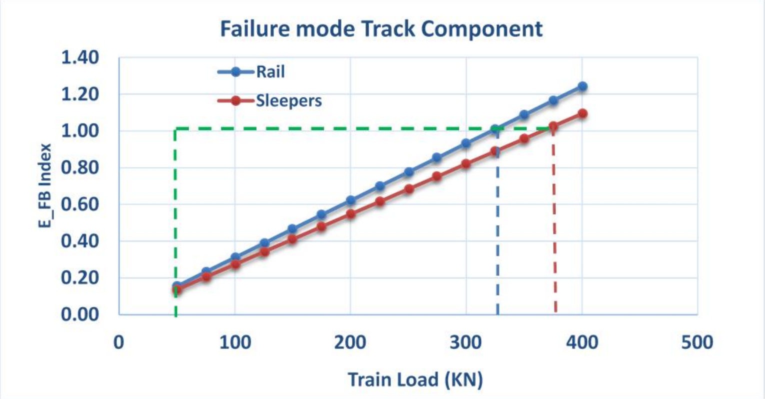

Fig. 7 Failure Discovery pattern for the Track components Springs

Failure segregation activities are hugely recommended as it grant the posibility to extract the exact unhealthy components from fault free once within a system or process. From table 4, the green zone expresses the healthy state of the rail and sleepers when put through acceptable train loading scenario. The unhealthy situations are gain under two distinct state of affair. The critical zone expressed in yellow portrays the domain at which the multiaxial stresses gain from the train loading nature is beyond the deformation strength (plastic zone) and the material nature of the said components carries its ultimate strength. The Grievous zone expressed in red showcases the domain at which the multiaxial stresses that encourage multiaxial fatigue damage are ready to portray fracture on respective track components. Additionally, the said component may either be possessing a True or Normal rapture strength. Meaning, when exploitation rolling stock track holding components good maintenance practices are required to stabilized a unique multiaxial fatigue limits within the green zone, in other to protect and encourage safety of freights, travellers, environments, sudden damage of infrastructure and personnel so as to minimize the cost of maintenance.

Table 4 Failure segregation matrix for Train Suspension system

Holding Components	Average working parameters	Equitable (0%)	Critical (10%)	Grievous (20%)
Rail	$(E_{FG}^T)_R$	**0.73 - 0.99**	**1**	> **1**
	$\left(\boldsymbol{\tau}_{\boldsymbol{hhmax}}^{\boldsymbol{T}}\right)_{\boldsymbol{R}}$	$\leq$ 1497.07 MPa	= 1621.83 MPa	= 1746.58 MPa
	$\left(\boldsymbol{\tau}_{\boldsymbol{hhmin}}^{\boldsymbol{T}}\right)_{\boldsymbol{R}}$	$\leq$ 1259.41 MPa	= 1357.86 MPa	= 1462.31 MPa
	F^T	< 325kN	= 325kN	> 325kN
Sleepers	$(E_{FG}^T)_{SP}$	**0.73 - 0.99**	**1**	> **1**
	$\left(\boldsymbol{\tau}_{\boldsymbol{hhmax}}^{\boldsymbol{T}}\right)_{\boldsymbol{is}}$	$\leq$ 1155.21 MPa	= 1237.77 MPa	> 666.66 MPa
	$\left(\boldsymbol{\tau}_{\boldsymbol{hhmin}}^{\boldsymbol{T}}\right)_{\boldsymbol{Sp}}$	$\leq$ 58.32 MPa	= 624.99MPa	> 1321.29MPa
	F^T	< 375kN	= 375kN	> 375kN
		Elastic Zone	Plastic Zone	Rupture

Reveal and online Storage Panel: these panels will display and stored historical input-output signals requested by the rolling stock agents and maintenance promoters for effortless follow up activities using numerical and linguistic terms, that expresses the nature of each component. Due to bulkiness and complexity, a unique strategy is still to be suggested for the design and construction of an intelligent electronic tool that grant the posibility to savegard and portrays an online reveal features developed to extract and follow-up the same attitude using hardware, software and wireless technologies. From table 4, the reveal panel is appraised during field validation where 0: $[(\%R_{FG})_i < 100\%]$ is considered as a maintenance free situation for each holding components and 1: $[(\%R_{FG})_i \geq 100\%]$ for possible repairs. Though the down time of a single components may lead to complete failure of the entired holding systems and set the rolling stock vehicle with other infrastructure in a risk zone (derailment). The duty of the savegard panel at this level is to memorized the historical signatures to be quantify using multiaxial fatigue stress criterion for individual components in strict accordance to their respective cycle counting index during field exploitation. These are going to be achieved from numerical field recording signatures gained from interconnected sensors that garantee the ability to quantify at every cycle counting the multiaxial stresses and the fatigue damage index with huge display using both offline/online iCloud technology.

V. CONCLUSION

These paper is aim at suggesting an intelligent strategy suitable to assist rolling stock operators to adequately improve on the design and maintenance of train-track holding systems subjected to multiaxial fatigue problems. Inclusively, a novel strategy that bring forth the use of model oriented technology centered on the Fogue fatigue criterion to aid diagnostic has been put to practice. The dynamic nature of rail and the sleepers under train changeable loading as the prime excitant has been examine with satisfaction. The variation of these estimated signals that depicts the nature of stresses within each component associated during field dynamics were opposed to clarify a unique diagnostic index through residual signals requested. To perform diagnostics using 0 for the healthy nature of the component and 1: for the presence of an unhealthy scenario. The remainder using model oriented and data oriented $(MB \neq DD)$ technologies are still to be showcased since the data oriented technique required historical signals gain from field evaluation. An intelligent electronics tools is still to be developed. For full upgrade on condition base maintenance plus for the rolling stock holding components, a strategy center on multiaxial fatigue life estimation is hugely requested since the follow-up of complex mechanical systems subdue to dynamic effects depends on proactive (prognostics) before active (diagnostics) phenomena in real time. The duty of the upcoming research will appraise the development of a proactive strategy, develop a sensor for multiaxial fatigue quantification, and also developed with field evaluation an online observation tool that is going to aid in improving the quality of maintenance activities.

REFERENCES

[1]. U.S Department of Transportation Statistic Report, (2021)

[2]. Liu, X., Rapik, S., M., Christopher, P., L.: Barkan, *Analysis of Causes of Major Train Derailment and Their Effect on Accident Rates. Journal of the Transportation Research Board, No. 2289,* Transportation Research Board of the National Academies, Washington, D.C., pp.154–163. DOI: 10.3141/2289-20 (2012).

[3]. Genel, K., Mehmet D., Turgut G.: *Corrosion fatigue behavior of ion nitride AISI4140 steel.* Material sciences and engineering, A 288 (2000) 91-100, Elsevier (2000).

[4]. Smith, R., A.: *Fatigue in Transport: Problems, Solutions and Future Threats.* Institution of Chemical Engineers Trans IChemE, Vol 76, Part B, pp 217-223 (1998).

[5]. Alfredo S., R., José A., F., O., Correia, António L., L., S., Abilío M., P., J.: *Evolution of Fatigue History.* Proceedings of COBEM 21st Congress of Mechanical Engineering, Natal, RN, Brazil (2011).

[6]. Japan Transport Safety Board Annual Report: *Railway accident and serious incident investigation,* 2016.

[7]. Selim, G., Melih, C., Seyid, F. D.: *Handbook of Materials Failure Analysis with Case Studies from the Aerospace and Automotive Industries*. doi.org/10.1016/B978-0-12-800950-5.00013-2, Elsevier (2016),

[8]. Dongchao, S., Zhuo Z., Changing L., Weidong W.: *The fatigue test and analysis for 50AT and 60AT railroad rails.* Damage & Fracture Mechanics VI, WIT Press, ISBN 1-85312-812 (2000).

[9]. Miroslav, N., Rostislav F., Bohuslav, R., R., R.: *Fatigue tests of railway axles.* Fatigue, Procedia Engineerin, 2259-2268, Science Direct (2010).

[10]. YAMAMOTO, M.: *Research and Development of Fatigue Issues for Railway Steel Products and Future Prospects.* Nippon Steel & Sumitomo Metal Technical Report No. 105 (2013).

[11]. Subash, G.: *Fabrication, Testing and Analysis of a Fatigue Sensor for Structural Health Monitoring.* Dissertations. 375. https://scholarworks.wmich.edu (2014).

[12]. .Jung-Won Seo, H., Hyun-Kyu J., Seok-Jin K., Dong-Hyeong L.: *Fatigue Design Evaluation of Railway Bogie with Full-Scale Fatigue Test.* Advances in Materials Science and Engineering Volume 2017, Article ID 5656497, 11 pages doi.org/10.1155/2017/5656497 (2017).

[13]. Jay, P., S., Prabir K., Vinayak R.: *Rolling Contact Fatigue Life of Rail for Different Slip Conditions*. Latin American Journal of Solids and Structures 14, 2243-2264 (2017).

[14]. Fang, X., Qi Y., Wenjie L., Wuming L., Rusong N., Huihao M.: Dynamic Stress of Subgrade Bed Layers Subjected to Train Vehicles with Large Axle Loads. Shock and Vibration, Volume 2018, Article ID 2916096, 12 pages, doi.org/10.1155/2018/2916096 (2018).

[15]. Guilherme, S., A., Gonçalo: *Fatigue Assessment of a High-Speed Railway Composite Steel-Concrete Bridge by The Hot-Spot Stress Method.* International Journal of Structural Integrity, DOI: 10.1108/IJSI-11-2017-0061(2018).

[16]. Chao, W., Chengqiang W., Chaotao L., Yongzhi J.: *Research on Fatigue Test Method of Car Body for High-speed trains.* IOP Conf. Series: Earth and Environmental Science 189 (2018) 062011 doi :10.1088/1755-315/189/6/062011 (2018)

[17]. Sołkowski, J.; Górszczyk, J.; Malicki, K.; Kudła, D.: *The Effect of Fatigue Test on the Mechanical Properties of the Cellular Polyurethane Mats Used in Tram and Railway Tracks*. Materials **2021**, 14, 4118. https://doi.org/10.3390/ma14154118 (2021).

[18]. Kenmeugne, B., D., Soh Fotsing, G., F., Anago, M., Fogue, J., L., Robert, J.-P. K., *On the evolution and comparison of multiaxial fatigue criteria*. International Journal of Engineering and Technology (2012).

[19]. Yingyu, W., Luca S.: *Critical Plane Approach to Multiaxial Variable Amplitude Fatigue Loading*. CORE, Fratturaed Integrità Strutturale, 33, 345-356; DOI: 10.3221/IGF-ESIS.33.38 (2015).

[20]. Bianzeube, T., Fabienne, F., Bastien, W., Jean-Louis R.: *A Multiaxial Variable Amplitude Fatigue Life Prediction Method Based on a Plane Per Plane Damage Assessment.* American Journal of Mechanical and Industrial Engineering, 3(4): 47-54, ISSN: 2575-6079 (Print); ISSN: 2575-6060 (Online), doi: 10.11648/j.ajmie.20180304.12 (2018).

[21]. Azevedo. D., Alberto, C., Bernardete, R.: *Estimation of Health Indicators using Advanced Analytics for Prediction of Aircraft Systems Remaining Useful Life time.* European Conference of the Prognostics and Health Management Society (2020).

[22]. Tse, S., A., Wolfgang, N., Bertin, S., Olivier, G., Tibi, B.: A Proposed Scheme for Fault Discovery and Extraction Using ANFIS: Application to Train Braking System. Studies in Engineering and Technology, ISSN 2330-2038 E-ISSN 2330-2046 (2020).

[23]. Sparthan, T., Nzie, W., Sohfotsing, B., Beda, T. Garro O.: A Valorized Scheme for Failure Prediction Using ANFIS: Application to Train Track Breaking System. Open Journalof Applied Sciences, 10, 732-757 (2020).

[24]. British-Steel-Rail-product-brochure, 2020

[25]. Arnold D., K.: On The Stress Analysis of Rails and Ties. Princeton University Department of Civil Engineering Princeton NJ 08540. Technical Report, 1976).