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Fuzzy Availability of a Pulping System used in the production of papers

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ABSTRACT
The production of paper is considered one of the important industries because papers are used daily for many purposes worldwide. This process is done in two stages. In the first stage, we add different types of materials and fibers to the raw material called wood pulp that is extracted from trees to control the physical properties of papers. In the second stage, the output of the first stage is entered into a paper-making machine to dry and suck moisture out of it. In this machine, there is a pulping system consists of four subsystems which are the digester, the knotter, the decker, and the opener. Knotter and Opener subsystems have two units, one is operating while another is kept in cold standby. In this paper, we will analyze the availability of that pulping system if failure and repair rates of the system units are exponentially distributed. Due to uncertainty and low of sufficient data, all parameters are fuzzy numbers represented by triangular-shaped membership functions. Finally, a numerical example is provided to illustrate the procedure used to determine the fuzzy availability of this pulping system.

KEYWORDS
Paper industry – standby - Markov process – Availability – fuzzy numbers.

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I. INTRODUCTION

To improve the quality of any product and create effective one satisfying the requirements of most customers, we consider two main goals which are the reduction of the product cost and the shortage of the design cycle time. For this reason, in the design phase, engineers analyze the reliability and the availability of different types of systems especially the large and complex ones. Reliability and availability are among the most important characteristics to be studied at the design stage of any product. The reliability of any system is defined as the probability that the system will perform its assigned function for a specified period under certain conditions. The availability is also defined as the probability that the system will perform its assigned function in a point of time (Lee, Chi Woo, et al. [1]). Large number of researchers introduced different repairable and unrepairable systems and calculate the reliability or the availability of these systems such as (Peng et al [2], Si, Wujun et al [3], Yang et al [4], El-Ghamry et al [5]).

In the real life, there is uncertainty and missing data related to the parameters of systems whose reliability or availability is required to be analyzed. For this reason, some researchers began to use the concept of fuzzy set which had first introduced by Zadeh [6] to represent these uncertain parameters after that they calculate the fuzzy reliability and the fuzzy availability of these systems. Elghamry et al. [7] analyzed the reliability of three-component chains and parallel systems with time-varying fuzzy failure rate. Agarwal et al. [8] studied the fuzzy availability of a crystallization system which used in the sugar production if this system consists of number of repairable units with fuzzy failure and repair rates. Also, Kumar and Goel [9] studied the fuzzy reliability of a pulping system which is used in the paper industry using general distributions of all random variables.

The production of paper is a complex industry depends on three major systems: a washing system, a feeding system, and a pulping system. Due to the importance of the repairable pulping system, in this paper we will present a new algorithm for analyzing its reliability when the failure and repair rates of its subsystems are constant with fuzzy parameters represented by triangular membership functions. This algorithm depends on $\alpha - cut$ technique so it is easier than the method introduced by Kumar and Goel [9] for calculating the fuzzy availability. Also, it can be applied to other complex fuzzy systems.

s_i:

This paper is arranged as follows: Section 2 introduces the definitions that will be used in the paper. Section 3presents the pulping system description, its components and assumptions used in the reliability analysis of that system. Section 4 explains how we can evaluate the availability of our system in the crisp case. Section 5 states the method used to get the fuzzy availability of our system. section 6 gives an illustrated numerical example. Finally, section 7 gives the conclusion.

II. Definitions

2.1 Fuzzy set Ã:

It is described as follows in a discourse world called X.

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\} \quad (1)$$

where, $\mu_{\tilde{A}}(x): X \to [0,1]$is the membership function which determines the grade of membership of any element $x \in$ X in the fuzzy set Ã and takes real values in the interval$[0,1]$.

2.2 α-Cut of fuzzy set $\tilde{A}$:

They are slices through the fuzzy set$\tilde{A}$producing regular crisp subsets of X. It can be written $\tilde{A}[\alpha]$ and defined as follow

$$\tilde{A}[\alpha] = [A_L(\alpha),\ A_U(\alpha)] = \{\, x \in\ X\ |\mu_{\tilde{A}}(x) \geq \alpha \,\}\ \ \forall\ \alpha \in [0,1] \quad (2)$$

2.3 Triangular fuzzy number $\tilde{A}$

It is denoted by $\tilde{A} = \langle a, b, c\rangle$and has membership function given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b < x \leq c \qquad \forall\ a < b < c \\ 0, & otherwise \end{cases} \quad (3)$$

With α-cut can be obtained by

$$\tilde{A}[\alpha] = [a + \alpha(b-a),\ \ c - \alpha(c-b)]\ \ \forall\ \alpha \in [0,1] \quad (4)$$

III. System description, notations, and assumption:

3.1 System description:

One of the most crucial components of the paper industry is the pulping system, which is made up of four subsystems connected in series. The **first subsystemA** is **digester** in whichthe chips are cooked for several hours using a steam heating system. Also, the failure of this subsystem causes the failure of the pulping system. The **second subsystemB** is **knotter** that eliminates the knots and consists of two units. One of the two units is in standby mode. when the two units in this subsystem fail, this subsystem fails.The **third subsystemC** is **decker** which containsthree deckers in series connection. Failure in any one of these deckers result in the failure of this subsystem. The **fourth Subsystem D** is **opener** that consists of two units. One rotates quickly and the other is on standby. Only when both units fail, our subsystem fails.

3.2 Notations

A, B, C, D :	is the operating states of the four subsystems.
$\bar{A}, \bar{B}, \bar{C}, \bar{D}$:	is the failed states of the four subsystems.
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$:	is the failure rates of the four subsystems.
$\mu_1, \mu_2, \mu_3, \mu_4$:	is the repair rates of the four subsystems.
$\tilde{A}(t)$:	is the fuzzy availability of the pulping system.
s_i:	is the system state at time t ; $i = 0,1,2, \ldots, 15$.
$\tilde{P}_i(t)$:	is the fuzzy probability of states_i at time t.
○:	demonstrates that subsystem is in full operation state.
◇:	demonstrates that subsystem is in standby mode state.
□:	demonstrates that subsystem is in failed state.

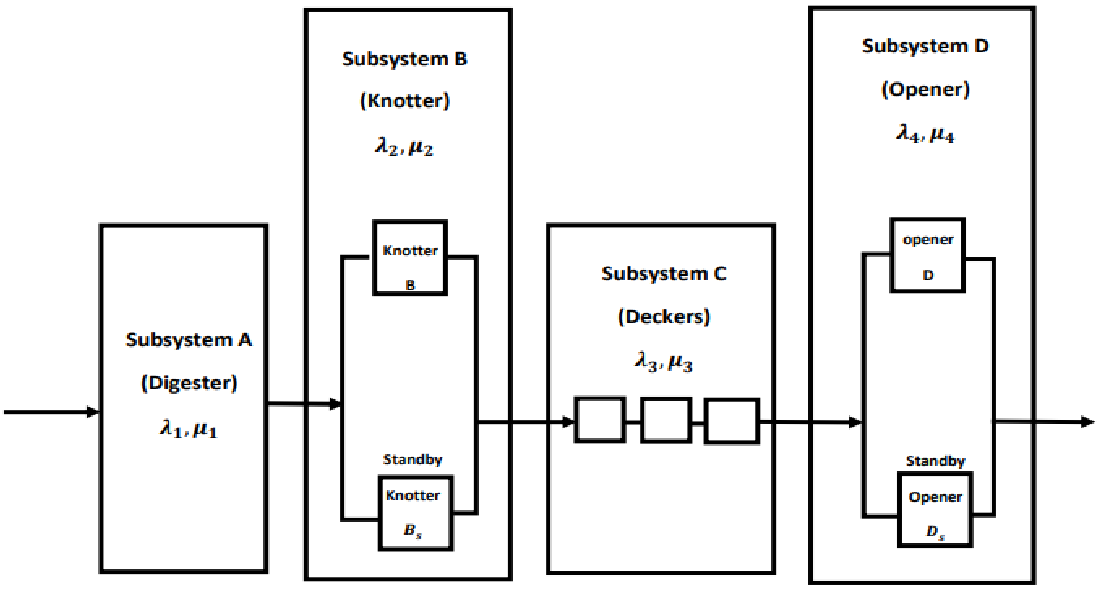

Figure.1: Block Diagram of the pulping system

3.3 Assumptions

Markov model of the pulping system is constructed to analyze its availability under the following assumptions:

- The system consists of four separate units: digester A, knotter B, decker C, and opener D connected in series.
- Each unit has two binary states, the working state, and the failed state.
- Both subsystems B and D have one unit in cold standby. When an active unit fails, one of standby units changes instantaneously to be active by perfect switching process.
- Failure time of standby or active unit follows Exponential distribution with parameter λ.
- When active or standby unit fails, it is immediately sent for repair to be as good as new.
- Any failed unit's repair time follows an exponential distribution with a parameter µ .

IV. Availability of the system in the CrispCase :

Based on the above description and assumptions, all possible states of our system are presented in Table.1.

The States of Markov Model of The Pulping System

Table.1

State Index "i"	System Status	Standby Units Status
0	working	B, D Cold standby
1	working	B Cold standby
2	working	D Cold standby
3	Working with full capacity	None of the cold standby units is in working status
$4 \leq i \leq 15$	Complete Failure	

Let $P_i(t)$; $i = 0,1,2,\ldots,15$ are the probabilities that the system is in the states S_i at any time t. From the state transition diagram shown in **Figure.2**, we can get system's first order differential equations in terms of the failure rates λ_1, λ_2, λ_3, λ_4 and the repair rates μ_1, μ_2,μ_3, μ_4 as follow:

$$\frac{dp_0(t)}{dt} = -(\mu_1 + \mu_3 + \lambda_2 + \lambda_4)p_0 + \mu_4 p_1 + \mu_2 p_2 + \lambda_1 p_5 + \lambda_3 p_6 \quad (5)$$

$$\frac{dp_1(t)}{dt} = -(2\mu_4 + \mu_1 + \mu_3 + \lambda_2)p_1 + \lambda_4 p_0 + \mu_2 p_3 + \lambda_3 p_7 + \lambda_1 p_8 + \lambda_4 p_9 \quad (6)$$

$$\frac{dp_2(t)}{dt} = \lambda_2 p_0 - (\mu_1 + 2\mu_2 + \mu_3 + \lambda_4)p_2 + \mu_4 p_3 + \lambda_1 p_4 + \lambda_2 p_{10} + \lambda_3 p_{11} \quad (7)$$

$$\frac{dp_3(t)}{dt} = \lambda_2 p_1 + \lambda_4 p_2 - (\mu_1 + 2\mu_2 + \mu_3 + 2\mu_4)p_3 + \lambda_2 p_{12} + \lambda_1 p_{13} + \lambda_4 p_{14} + \lambda_3 p_{15} \quad (8)$$

$$\frac{dp_4(t)}{dt} = \mu_1 p_2 - \lambda_1 p_4 \quad (9) \qquad \frac{dp_5(t)}{dt} = \mu_1 p_0 - \lambda_1 p_5 \quad (10)$$

$\frac{dp_6(t)}{dt} = \mu_3 p_0 - \lambda_3 p_6$	(11)	$\frac{dp_7(t)}{dt} = \mu_3 p_1 - \lambda_3 p_7$	(12)
$\frac{dp_8(t)}{dt} = \mu_1 p_1 - \lambda_1 p_8$	(13)	$\frac{dp_9(t)}{dt} = \mu_4 p_1 - \lambda_4 p_9$	(14)
$\frac{dp_{10}(t)}{dt} = \mu_2 p_2 - \lambda_2 p_{10}$	(15)	$\frac{dp_{11}(t)}{dt} = \mu_3 p_2 - \lambda_3 p_{11}$	(16)
$\frac{dp_{12}(t)}{dt} = \mu_2 p_3 - \lambda_2 p_{12}$	(17)	$\frac{dp_{13}(t)}{dt} = \mu_1 p_3 - \lambda_1 p_{13}$	(18)
$\frac{dp_{14}(t)}{dt} = \mu_4 p_3 - \lambda_4 p_{14}$	(19)	$\frac{dp_{15}(t)}{dt} = \mu_3 p_3 - \lambda_3 p_{15}$	(20)

With the normalization condition, $\sum_{i=0}^{15} p_i(t) = 1$ (21)

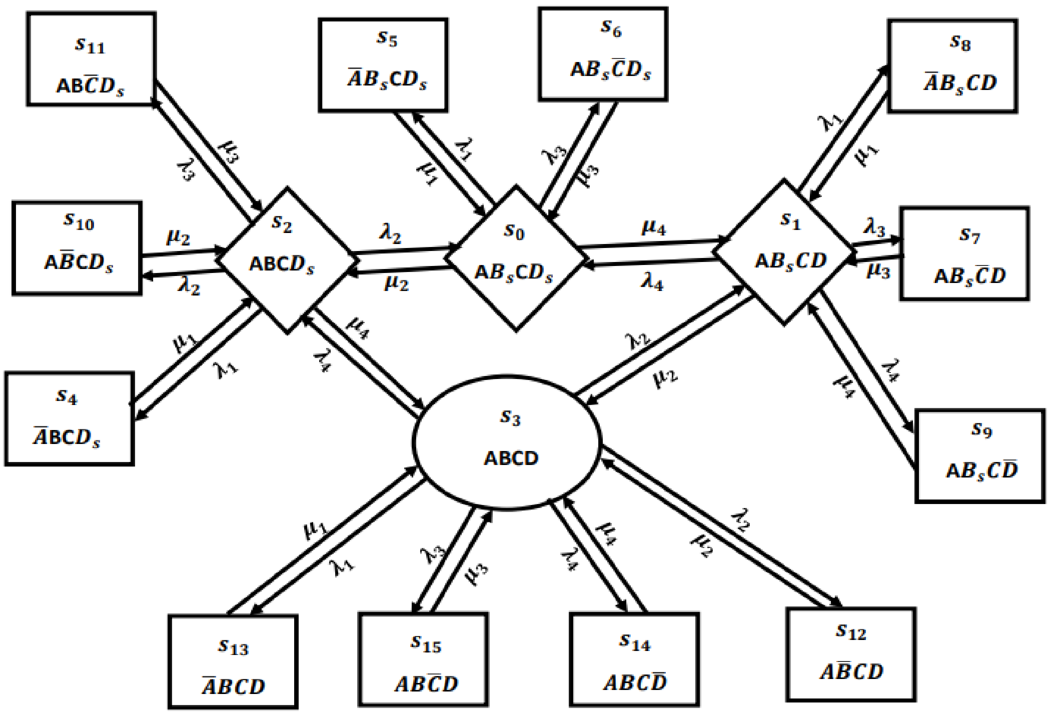

Figure.2: State Transition Diagram for Availability Analyses of The Pulping System

V. Availability of Fuzzy System

For more realism, weconsider the failure rates λ_1,λ_2,λ_3,λ_4 and the repair rates μ_1,μ_2,μ_3,μ_4 of our model are fuzzynumbers$\widetilde{\lambda_1}$, $\widetilde{\lambda_2}$,$\widetilde{\lambda_3}$,$\widetilde{\lambda_4}$,$\widetilde{\mu_1}$,$\widetilde{\mu_2}$,$\widetilde{\mu_3}$ and$\widetilde{\mu_4}$ with triangular shaped membership functions obtained from experts or estimated using statistical data. The fuzzy availability of our model can be computed by using the α – cut technique with the following procedures:

Step 1: We substitute in the system of equations (5 - 21) by the crisp intervals ($\alpha - cuts$) of the fuzzy parameters $\widetilde{\lambda_1}$, $\widetilde{\lambda_2}$,$\widetilde{\lambda_3}$,$\widetilde{\lambda_4}$,$\widetilde{\mu_1}$,$\widetilde{\mu_2}$,$\widetilde{\mu_3}$ and $\widetilde{\mu_4}$which are obtained at distinct values of ; $\alpha \in [0, 1]$.

Step 2: Solve simultaneously these equations (5 - 21) by using Maple program under the initial conditions (22) to obtain $\alpha - cuts$ of fuzzy transition probabilities $\tilde{P}_i(t)$; $i = 1, 2, \ldots, 15$.

Step 3: Calculate $\alpha - cuts$ of system fuzzy availability function $\tilde{A}(t)$ at any time t corresponding to the chosen values of α by using relation (23).

VI. Numerical Example

we will use the algorithm introduced in the previous section to get the fuzzy availability function of our system if the failure rates represented by triangular membership functions$\widetilde{\lambda_1} =< 0.00635, 0.00675, 0.00690 >$, $\widetilde{\lambda_2} =< 0.00121, 0.00148, 0.00169 >$, $\widetilde{\lambda_3} =< 0.0039, 0.0065, 0.0080 >$, $\widetilde{\lambda_4} =< 0.00759, 0.00815, 0.00868 >$ and the repair rates are $\widetilde{\mu_1} =< 0.0945, 0.0975, 0.099 >$, $\widetilde{\mu_2} =< 0.145, 0.165, 0.177 >$, $\widetilde{\mu_3} =< 0.0284, 0.0295, 0.0310 >$, $\widetilde{\mu_4} =< 0.0452, 0.0477, 0.04927 >$.

We can first obtain the $\alpha - cut$ of all these triangular fuzzy numbers as follow

$[\lambda_1{}^L(\alpha), \lambda_1{}^U(\alpha)] = [0.00635 + 0.0004\alpha, 0.00690 - 0.00015\alpha]$
$[\lambda_2{}^L(\alpha), \lambda_2{}^U(\alpha)] = [0.00121 + 0.00027\alpha, 0.00169 - 0.00021\alpha]$
$[\lambda_3{}^L(\alpha), \lambda_3{}^U(\alpha)] = [0.0039 + 0.0026\alpha, 0.0080 - 0.0015\alpha]$
$[\lambda_4{}^L(\alpha), \lambda_4{}^U(\alpha)] = [0.00759 + 0.00056\alpha, 0.00868 - 0.00053\alpha]$
$[\mu_1{}^L(\alpha), \mu_1{}^U(\alpha)] = [0.0945 + 0.003\alpha, 0.0990 - 0.0015\alpha]$
$[\mu_2{}^L(\alpha), \mu_2{}^U(\alpha)] = [0.145 + 0.02\alpha, 0.177 - 0.012\alpha]$
$[\mu_3{}^L(\alpha), \mu_3{}^U(\alpha)] = [0.0284 + 0.0011\alpha, 0.0310 - 0.0015\alpha]$
$[\mu_4{}^L(\alpha), \mu_4{}^U(\alpha)] = [0.0452 + 0.0025\alpha, 0.0492 - 0.0015\alpha]$

After solving our model's equations (5 - 21) numerically by using MATLAB program under the initial conditions (22) at arbitrary values of α, we obtain the fuzzy transition probabilities $\tilde{P}_i(t)$; $i = 0, 1, 2, \dots, 15$. Next, we substitute in relation (23) to get the fuzzy availability $\tilde{A}(t)$ of our system with time t at the chosen values of α and represent it graphically as shown in **Figure.3**. Also, this fuzzy availability can be represented at any instant value of time (take $t = 0.3$) as shown in **Figure.4.**

From the results obtained in this example, we observe that the fuzzy availability of our system at any time t can be approximated to be triangular fuzzy number as the fuzzy failure and repair rates of the system units. In addition, the graph of the fuzzy availability function was not a one curve as usual but it had two curves at each value of α; $\alpha \in [0, 1]$ that represent the lower and the upper bounded curves. All curves obtained at different values of αwere around a main curve evaluated at $\alpha = 1$ (crisp case) and were bounded by two curves evaluated at $\alpha = 0$.

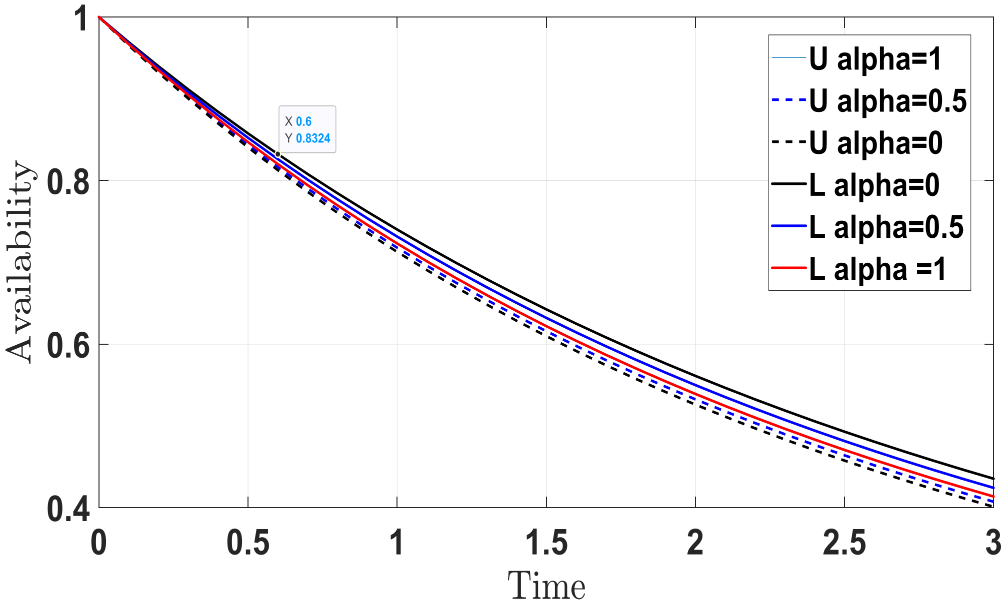

Figure.3: Fuzzy Availability function versus the time for the pulping systemat $\alpha - cut = 0, 0.5,$ and 1

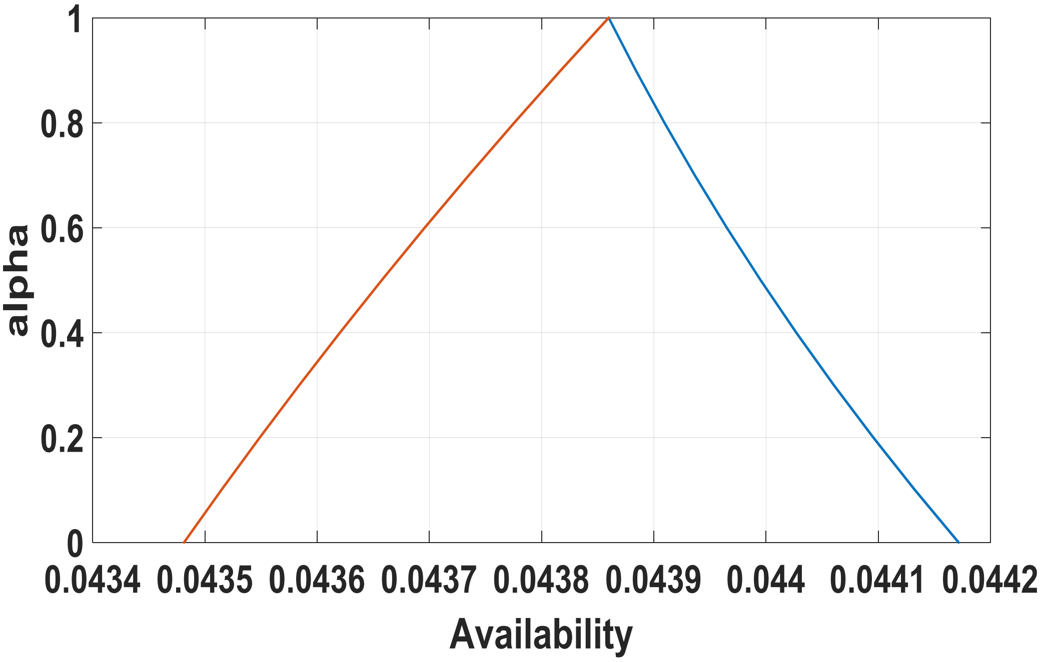

Figure.4: Fuzzy Availability function of the pulping systemat $t = 0.3$

VII. Conclusion

In this research, we propose a model of a repairable pulping system which plays an important role in the industry of paper. This system consists of four subsystems digester, knotter, decker, and opener. In the knotter and opener subsystems, there is one unit in operation, and another is kept in cold standby. we assume the time to failure and the time to repair of each unit are exponentially distributed. The knowledge about the system parameters is usually described with large uncertainty content so the classical approach based on the probability theory became inappropriate for analyzing the availability of our system. For this reason, these parameters are considered fuzzy numbers with triangular membership functions. The analysis procedures, based on the $\alpha -$ cut concept, are used to evaluate the system fuzzy availability and a numerical example is given to illustrate the performance of our model and evaluate the fuzzy availability with graphs by using MATLAB software program.

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