American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-11, Issue-09, pp-55-67
www.ajer.org
Research Paper Open Access

Features of the TE_{01} Wave Excitation in a Composite Open Resonator

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ABSTRACT: *The paper deals with the excitation efficiency η of the TE_{01} wave in a circular waveguide made in the center of one of the mirrors with the help of higher TEM_{30q} and TEM^{*}_{11q} modes of such a composite open resonator (OR). It is shown that the excitation efficiency of this wave by the indicated modes of resonator is not high. If the TE_{01} wave is excited using the inner ring of the field of the TEM^{*}_{11q} mode, the value η increases to 95.5% at the radius of the circular waveguide a equal to $0.993w_0$. The experimental studies carried out have confirmed that the TE_{01} wave is excited in the waveguide with high efficiency. The presence of a round oversized waveguide section increases the losses in the OR only by 0.9 dB. The test body method was used to measure the distributions of the fields of the TEM_{3012} and TEM^{*}_{1112} modes in a hemispherical OR, which consists of totally reflecting mirror in one case and contains a section of a round oversized waveguide in the center in the other case. The waveguide turned out to cause the transformation of the highest axially asymmetric TEM_{3012} mode into a degenerate axially symmetric TEM^{*}_{1112} mode. In this case, the electric field structure in the inner ring of the field of the mode under consideration becomes similar to the electric field structure of the TE_{01} wave in the waveguide. The composite OR under investigation can be used as a storage resonator in the construction of electromagnetic pulse compressors in the EHF range.***

KEYWORDS: *open resonator, mode, excitation efficiency, loaded Q-factor, round waveguide, field structure, power compressor.*

Date of Submission: 01-09-2022 Date of acceptance: 13-09-2022

I. INTRODUCTION

To study the interaction of an electromagnetic field with matter, electronic devices and biological objects, powerful sources of single and repetitively pulsed microwave radiation of nanosecond duration are needed. One of the ways to obtain powerful microwave pulses is the method of resonant pulse compression [1-3]. It is based on the slow accumulation and rapid removal of electromagnetic energy from a high-q resonator. This method makes it possible to use standard generators with a relatively low output power level and a long (microsecond) pulse duration to obtain short microwave pulses. At the same time high peak power and an ability to work with a high repetition rate make microwave compressors very attractive for solving a wide range of problems.

Single-mode cavity resonators based on waveguide *H*-tees are used to accumulate microwave energy in the centimeter (cm) range. Interference switches which are electrically controlled or operating on self-breakdown arresters are used to output energy [4,5]. A significant disadvantage of such compressors is the low q-factor of the storage resonator. Therefore, compressors have been developed in this range with the use of oversized waveguide resonators with axial symmetry, operating on the TE_{0n} mode. Due to the abnormally small losses of such modes, it is possible to increase the q-factor of the resonant system significantly.

Microwave compressors of the first and second types have large geometric dimensions. The length of such a compressor exceeds 2 m in the 3 cm range. Voltage with a pulse amplitude of several tens of kV and higher is required for the normal operation of an electric spark gap. To reduce geometrical dimensions of the compressors and decrease the surge voltage of the arrester it is necessary to switch to the extremely high frequency (EHF) range.

The geometric dimensions of the resonant systems decrease in this range, but the ohmic losses increase. Hence, to obtain large values of the q-factor, which is a necessary condition for the creation of compressors, using resonant systems adequate to this range, viz. open resonators (OR), is required. The geometric dimensions of the OR are a few dozens of wavelengths. The connection with free space in such resonators provides additional selection of the oscillation spectrum and free access to the resonant volume. In [6], a compressor in the form of a three-mirror resonator excited by a Gaussian beam was tested at a low power level at a frequency of 34.27 GHz. Each mirror diameter of the traveling wave resonator is 227 mm. This compressor dimensions and the complexity of mirror alignment do not allow it to be used for solving practical problems. Therefore, it is desirable to include the OR in the waveguide transmission line. This is due to a simpler way of modes excitation in the resonant volume. Slotted coupling elements are used here for the input and output of energy. They represent smooth transitions from a reduced section to a standard section of a rectangular waveguide. It is impossible to withdraw energy from the resonator in a short period of time through such coupling elements. Thus, it is necessary to use segments of oversized waveguides [7-9] to extract energy from the OR. There is a degenerate axisymmetric TEM^*_{01q} mode in the resonator in [10]. A TE_{01} wave is excited in a section of a round oversized waveguide made in the center of a flat mirror OR. The diameter of this waveguide is definitely related to the geometry of the resonator and the wavelength λ. The curvature radius of the spherical mirror and the distance between the mirrors should be increased to raise the Q-factor of the TEM^*_{01q} mode. In this case, the diameter of the circular waveguide segment will have such dimensions that the indicated mode will not be excited in the resonator.

In order to reduce the diameter of a circular waveguide section, it is necessary to move on to higher modes of OR. The higher the transverse index of the mode, the narrower it is to the resonator axis [11]. Therefore, with the geometric dimensions of the OR, providing a high Q-factor of the highest mode, the diameter of the circular waveguide with the TE_{01} wave must have acceptable dimensions for building a power compressor in the EHF range.

Thus, the objective of this work is to study the features of the TE_{01} wave excitation in a circular waveguide included in the composition of the OR, using two higher TEM_{30q} and TEM^*_{11q} modes of the resonant system.

II. EFFICIENCY OF TE_{01} WAVE EXCITATION IN A CIRCULAR WAVEGUIDE USING TEM_{30q} AND TEM^*_{11q} MODES OR

Consider the excitation of a higher axially symmetric TE_{01} wave in a circular waveguide of radius a by means of a higher axially asymmetrical TEM_{30q} mode OR. The geometric parameters of the resonator under consideration and the distribution of the exciting $\vec{E}_e(x,y)$ and $\vec{E}_w(x,y)$ working fields are shown in Fig. 1. Waveguide 3 is made in the center of flat mirror 1 of the hemispherical OR. The resonant length of this waveguide is determined by the position of piston 4. The given mode is assumed to be excited in the resonator from the side of spherical mirror 2.

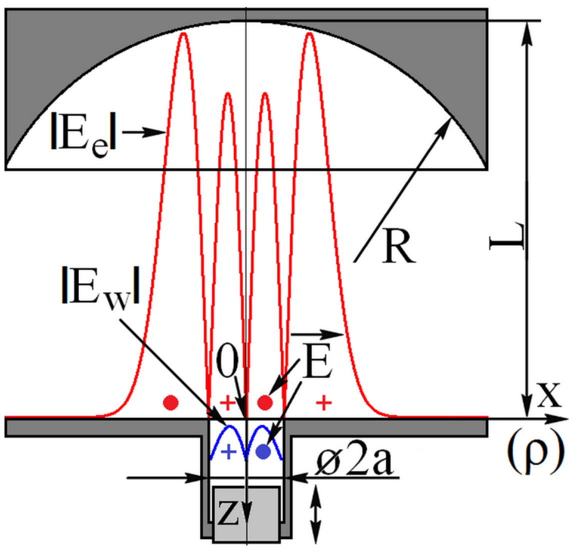

Figure 1: Hemispherical OR

The figure also shows the orientation of the electric field strength vectors in "field spots" of the TEM_{30q} mode and in the TE_{01} guided wave. Let us write the distribution of the field electric component of the TEM_{30q} mode in the plane z=0 in a cylindrical coordinate system, taking into account the vector nature of the electric field [12]

$$\vec{E}_e(\rho,\varphi)=4E_0\left(\sqrt{2}\rho/w_0\right)\exp\left(-\rho^2/w_0^2\right)\left[2\left(\sqrt{2}\rho\cos\varphi/w_0\right)^2-3\right]\left(\vec{\rho}_0\frac{1}{2}\sin 2\varphi+\vec{\varphi}_0\cos^2\varphi\right). \quad (1)$$

Here, E_0 - is the amplitude coefficient, w_0 - is the spot field radius of the main TEM_{00q} mode of the resonator on flat mirror 1, $\vec{\rho}_0$ and $\vec{\varphi}_0$ are basic vectors. The distribution of the electrical component of the TE_{01} wave field in a circular waveguide in the plane z=0 has the form of:

$$\vec{E}_w(\rho,\varphi)=(C_0/N_{01})\kappa_{01}(-J_1(\kappa_{01}\rho))\vec{\varphi}_0, \quad (2)$$

where $C_0=ik_0W_0\mu_0$, $k_0=\omega\sqrt{\varepsilon_0\mu_0}$, $W_0=\sqrt{\mu_0/\varepsilon_0}=120\pi$, N_{01} – normalization constant, $\kappa_{01}=3.832/a$ – transverse

wave number, a – radius of a circular waveguide, $J_1(\kappa_{01}\rho)$ – Bessel function of the first kind. The reflection from the waveguide aperture is neglected. As shown in [7], in order to excite the TE_{01} wave in a circular waveguide with maximum efficiency, its diameter must exceed several wavelengths. With such geometric dimensions of the aperture, reflection from it can be neglected [13]. The apertures of the mirrors are considered to be infinite.

To determine the excitation efficiency of the TE_{01} wave in a circular waveguide with the help of the TEM_{30q} mode, we use representations (1), (2) and the ratio of [14]

$$\eta=\left|\int_0^a\int_0^{2\pi}\vec{E}_e(\rho,\varphi)\vec{E}_w^*(\rho,\varphi)\rho\, d\rho\, d\varphi\right|^2\Bigg/\left(\left\|\vec{E}_e(\rho,\varphi)\right\|^2\left\|\vec{E}_w(\rho,\varphi)\right\|^2\right). \quad (3)$$

The symbol *) denotes the complex conjugate function. In fact, this ratio shows what part of the energy stored in the TEM_{30q} mode of the resonator is converted into the energy of the TE_{01} wave of the circular waveguide.

$\left\|\vec{E}_e(\rho,\varphi)\right\|^2$ and $\left\|\vec{E}_w(\rho,\varphi)\right\|^2$ values are the squares of the function norms of the exciting and working fields. They are defined by the expressions [14]

$$\left\|\vec{E}_e(\rho,\varphi)\right\|^2=\int_0^\infty\int_0^{2\pi}\vec{E}_e(\rho,\varphi)\,\vec{E}_e^*(\rho,\varphi)\rho d\rho d\varphi, \quad (4)$$

$$\left\|\vec{E}_w(\rho,\varphi)\right\|^2=\int_0^a\int_0^{2\pi}\vec{E}_w(\rho,\varphi)\,\vec{E}_w^*(\rho,\varphi)\rho d\rho d\varphi. \quad (5)$$

$\vec{E}_e(\rho,\varphi)$ and $\vec{E}_w(\rho,\varphi)$ functions are described by expressions (1) and (2) respectively. After substituting their values into expressions (4), (5) and omitting intermediate calculations, the final form is obtained

$$\left\|\vec{E}_e(\rho,\varphi)\right\|^2=24\pi E_0^2 w_0^2, \quad (6)$$

$$\left\|\vec{E}_w(\rho,\varphi)\right\|^2=\left(C_0^2/N_{01}^2\right)\kappa_{01}^2 2\pi a^2\int_0^1 u\left(J_1(3.832u)\right)^2 du. \quad (7)$$

After substituting the values determined by expressions (1), (2), (6) and (7) into relation (3) and omitting intermediate calculations, the final form of the expression determining the excitation efficiency of the TE_{01} guided wave in a circular waveguide using a higher axially asymmetrical TEM_{30q} mode of the resonator is obtained

$$\eta(\tilde{a})=6\tilde{a}^4\frac{\left|\int_0^1 u^2\exp\left(-u^2\tilde{a}^2\right)J_1(3.832u)du-\tilde{a}^2\int_0^1 u^4\exp\left(-u^2\tilde{a}^2\right)J_1(3.832u)du\right|^2}{\int_0^1 u\left(J_1(3.832u)\right)^2du}, \quad (8)$$

where $\tilde{a}=a/w_0$. The dependence of the $\eta(\tilde{a})$ on the normalized radius of a circular waveguide $\tilde{a}$ is shown in Fig. 2 (curve 1). Under the consideration of the above we have confined ourselves to the value of $\tilde{a}$ = 2. It corresponds to the radius of the circular waveguide a equal to the field spot diameter ($2w_0$) of the fundamental TEM_{00q} mode of the resonator on a flat mirror. It is not advisable to increase the diameter of a circular waveguide. With the increase of $\tilde{a}$, the excitation efficiency of the TE_{01} wave with the help of the considered mode of OR will decrease. In this case, the excitation efficiency of the TE_{02} wave of the round waveguide will increase, since its structure is the closest one to the structure of the field of the TEM_{30q} mode in the resonator. However, as mentioned above, the excitation of the TE_{01} wave in a circular waveguide is under consideration.

With a relatively small radius of a circular waveguide ($\tilde{a} < 2$) made in the center of a flat mirror, the "field spots" of the TEM_{30q} mode OR (Fig. 1) by the edge should be located on the mirror surface without getting into the aperture of the circular waveguide. Fig. 2 (curve 1) demonstrates that the maximum value η^{max} =0.1212 at $\tilde{a} = 0.993$. Thus, the maximum excitation efficiency of the considered guided wave with the help of the higher axially asymmetric TEM_{30q} mode of the OR is low.
Find the second zero of the function (1), which describes the distribution of the electric field component of the TEM_{30q} mode in the OR. The first zero of the function considered takes place at $\rho_1 = 0$. Assuming that $\varphi = 0$ and equating the expression (1) to zero, we get $\rho_3 = w_0\sqrt{3}/2$ or $\tilde{\rho}_3 = \rho_3/w_0 = 0.866$. As shown, the maximum excitation efficiency of the TE_{01} wave takes place in the case when the electric node of the TEM_{30q} mode is not located at the edge of the round waveguide, but is slightly shifted into its aperture.

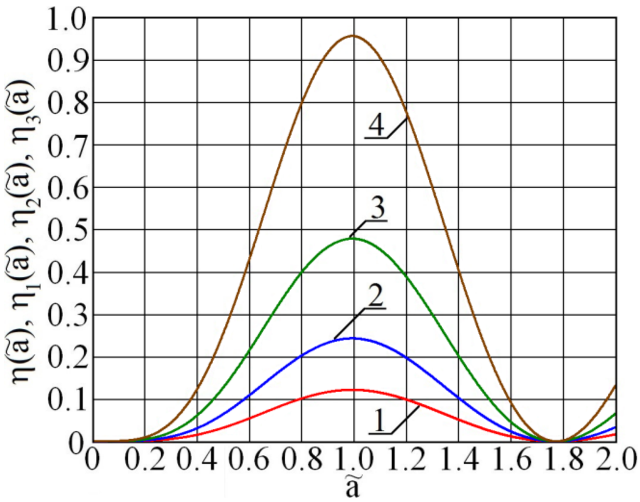

Figure 2: The excitation efficiency of the TE_{01} wave in a circular waveguide using the highest TEM_{30q} and TEM^*_{11q} modes OR

Now suppose that the resonator has a higher degenerate axially symmetrical TEM^*_{11q} mode [10]. This mode, in fact, is a superposition of two TEM_{30q} and TEM_{03q} modes mixed in space, and their phases are shifted by 90° [15]. In this case, the distribution of the electric field component of the TEM^*_{11q} mode in a plane *z*=0 in a cylindrical coordinate system, taking into account the vector nature of the electric field and transformations performed, has the form of

$$\vec{E}_e(\rho,\varphi) = E_0\left(\sqrt{2}\rho/w_0\right)\exp\left(-\rho^2/w_0^2\right)\left\{2\left(\sqrt{2}\rho/w_0\right)^2 \sin 4\varphi\,\vec{\rho}_0 +.\right.$$
$$\left. + \left[2\left(\sqrt{2}\rho/w_0\right)^2 \cos 4\varphi + 6\left(\sqrt{2}\rho/w_0\right)^2 - 12\right]\vec{\varphi}_0\right\}. \quad (9)$$

The distribution of the electric field component of the TE_{01} wave in a circular waveguide and the square of the function norm describing the working field $\vec{E}_w(\rho,\varphi)$, as it was in the previous case, will be described by expressions (2) and (7) respectively. In this case, the square of the function norm $\left\|\vec{E}_e(\rho,\varphi)\right\|^2$ which describes the exciting field $\vec{E}_e(\rho,\varphi)$, taking into account expressions (4) and (9), will be determined by the equation of

$$\left\|\vec{E}_e(\rho,\varphi)\right\|^2 = 48\pi E_0^2 w_0^2. \quad (10)$$

After substituting the values $\vec{E}_e(\rho,\varphi)$, $\vec{E}_w(\rho,\varphi)$, $\left\|\vec{E}_e(\rho,\varphi)\right\|^2$ and $\left\|\vec{E}_w(\rho,\varphi)\right\|^2$, determined by (9), (2), (10) and (7) into relation (3), the excitation efficiency $\eta_1(\tilde{a})$ of the TE_{01} wave in a circular waveguide is obtained using the axially symmetric TEM^*_{11q} mode OR.

$$\eta_1(\tilde{a}) = 12\tilde{a}^4 \frac{\left| \int_0^1 u^2 \exp\left(-u^2\tilde{a}^2\right) J_1(3.832u) du - \tilde{a}^2 \int_0^1 u^4 \exp\left(-u^2\tilde{a}^2\right) J_1(3.832u) du \right|^2}{\int_0^1 u \left(J_1(3.832u)\right)^2 du}. \qquad (11)$$

As can be seen, the excitation efficiency $\eta_1(\tilde{a})$ of the TE_{01} wave in a circular waveguide of radius *a* doubled in this case compared to the previous case [see expression (8)], i.e. $\eta_1(\tilde{a}) = 2\eta(\tilde{a})$. The maximum excitation efficiency of this guided wave is $\eta_1^{\max}(\tilde{a})$ = 0.2424 at $\tilde{a} = 0.993$ (Fig. 2, curve 2). Here, as in the previous case, the value $\tilde{a}$ is to be lower than 2. The obtained value $\eta_1^{\max}(\tilde{a})$ shows that the excitation efficiency of the TE_{01} wave in a circular waveguide with the help of the axially symmetric TEM_{11q}^* mode of the OR has increased, but only slightly.

As the next step, the following assumption should be made. A wave in a circular waveguide can be considered to be excited not by the whole TEM_{30q} mode OR defined by expression (1), but by means of two central "field spots" (Fig. 1). The fact that at a certain value $\rho = \rho_3$ $E/E^{\max} = 0$ allows to draw such a conclusion. In this case, when calculating the square of the function norm of the exciting field $\left\| \vec{E}_e(\rho,\varphi) \right\|^2$ defined by expression (4), we will integrate in a cylindrical coordinate system from zero to the value $\rho_3 = w_0\sqrt{3}/2$. This value, as mentioned above, is the second zero of the function $\vec{E}_e(\rho,\varphi)$ describing the distribution of the electric field component of the TEM_{30q} mode on the flat mirror of the hemispherical OR. The distribution of the electrical field component of the TE_{01} wave in a circular waveguide and the square of the function norm describing the working field $\vec{E}_w(\rho,\varphi)$, as in the previous cases, will be determined by expressions (2) and (7). Omitting intermediate steps of this computation, the final form is written

$$\left\| \vec{E}_e(\rho,\varphi) \right\|^2 = \frac{32}{w_0^2} \pi E_0^2 a^4 \int_0^{\sqrt{3}/2\tilde{a}} u^3 \left(10u^4\tilde{a}^4 - 18u^2\tilde{a}^2 + 9\right)^2 \exp\left(-2u^2\tilde{a}^2\right) du. \qquad (12)$$

Now, using expressions (1), (2), (7) and (12), from the relation (3) we obtain an expression that determines the efficiency of excitation of the TE_{01} wave in a circular waveguide by means of two central "field spots" of the TEM_{30q} mode of the OR under consideration (Fig. 1). Omitting intermediate calculations, the final form must be presented as

$$\eta_2(\tilde{a}) = \frac{4.5\left| \int_0^1 u^2 \exp\left(-u^2\tilde{a}^2\right) J_1(3.832u) du - \tilde{a}^2 \int_0^1 u^4 \exp\left(-u^2\tilde{a}^2\right) J_1(3.832u) du \right|^2}{\int_0^{\sqrt{3}/2\tilde{a}} u^3 \left(10u^4\tilde{a}^4 - 18u^2\tilde{a}^2 + 9\right)^2 \exp\left(-2u^2\tilde{a}^2\right) du \int_0^1 u \left(J_1(3.832u)\right)^2 du}. \qquad (13)$$

The results of the calculation using the formula (13) are shown in Fig. 2 (curve 3). It is easy to see from the figure that the maximum excitation efficiency of the TE_{01} wave in this case has increased by more than four times compared to the first case (curve 1) and totaled $\eta_2^{\max}(\tilde{a})$ = 0.4774 at $\tilde{a}$ = 0.993.

Now consider the excitation of the guided wave in question with the help of the central ring of the degenerate TEM_{11q}^* mode, defined by expression (9). When calculating the squared function norm of the exciting field $\left\| \vec{E}_e(\rho,\varphi) \right\|^2$, integrating in a cylindrical coordinate system from zero to the value $\rho_3 = w_0\sqrt{3}/2$ is to be carried out . From expression (4) we obtain

$$\left\| \vec{E}_e(\rho,\varphi) \right\|^2 = \frac{64}{w_0^2} \pi E_0^2 a^4 \int_0^{\sqrt{3}/2\tilde{a}} u^3 \left(10u^4\tilde{a}^4 - 18u^2\tilde{a}^2 + 9\right)^2 \exp\left(-2u^2\tilde{a}^2\right) du. \qquad (14)$$

Here, as above, $\vec{E}_w(\rho,\varphi)$ and $\left\| \vec{E}_w(\rho,\varphi) \right\|^2$ will be described by expressions (2) and (7). Now, using expressions (9), (2), (14), (7) and (3), the final form of the ratio determining the excitation efficiency of the TE_{01} wave with the help of the central ring of the TEM_{11q}^* mode OR is written down.

$$\eta_3(\tilde{a}) = \frac{9\left| \int_0^1 u^2 \exp\left(-u^2\tilde{a}^2\right) J_1(3.832u) du - \tilde{a}^2 \int_0^1 u^4 \exp\left(-u^2\tilde{a}^2\right) J_1(3.832u) du \right|^2}{\int_0^{\sqrt{3}/2\tilde{a}} u^3 \left(10u^4\tilde{a}^4 - 18u^2\tilde{a}^2 + 9\right)^2 \exp\left(-2u^2\tilde{a}^2\right) du \int_0^1 u \left(J_1(3.832u)\right)^2 du}. \qquad (15)$$

As it turns out, in this case $\eta_3(\tilde{a}) = 2\eta_2(\tilde{a})$. The results of calculating the excitation efficiency $\eta_3(\tilde{a})$ of the TE_{01} wave in a circular waveguide using the central part of the TEM^*_{11q} mode OR are shown in Fig. 2 (curve 4). It can be seen from the figure that the excitation efficiency of the TE_{01} wave in a circular waveguide with the help of the central ring of the TEM^*_{11q} mode OR has increased significantly. At the same time, its maximum value has doubled in comparison with the previous case and totaled $\eta_3^{\max}(\tilde{a})$=0.9548 at $\tilde{a}$ =0.993. In order to verify the correctness of our assumptions when calculating the excitation efficiency of the TE_{01} wave in a circular waveguide made in the center of one of the OR mirrors, it is necessary to carry out experimental studies.

III. DESCRIPTION OF THE EXPERIMENTAL STAND

The block diagram of the experimental stand, which was used to study the features of the excitation of the higher axially symmetric TE_{01} wave in a round waveguide, made in the center of a flat mirror of a hemispherical OR, is shown in Fig. 3. The influence of this segment of the circular waveguide on the electric field structure of the highest TEM_{30q} mode OR was analyzed with the help of that stand. Fig. 4 shows the appearance of the experimental stand.

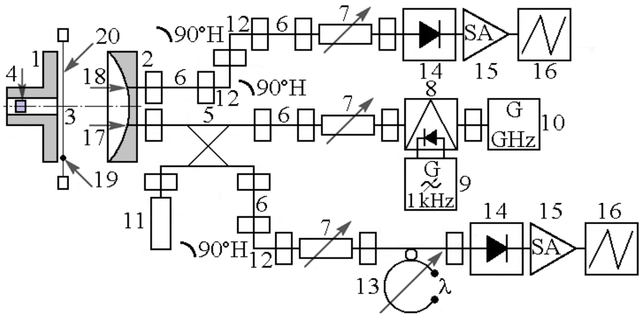

Figure 3: Block diagram of the experimental stand

The hemispherical OR is formed by a flat mirror 1 with an aperture of 59 mm and a spherical focusing mirror 2 with a curvature radius of *R*=41 mm and the same aperture. The mirrors have been produced of D16T duralumin. Inserts made of brass with a diameter of 10 mm have been pressed into the spherical mirror 2 at the locations of the coupling elements 17 and 18. This is done in order to be able to solder slotted coupling elements. A segment of a round waveguide 3 with a diameter of 10 mm has been made in the center of the flat mirror (Fig. 3, 5). The explanation why waveguide 3 has such a diameter will be given below. The resonant length of this waveguide segment can be adjusted using piston 4. The resonant length of this waveguide segment can be adjusted by moving piston 4 by means of a micrometer screw. Since the contact of the piston with the waveguide walls is not necessary for the TE_{01} wave, its diameter is 9.5 mm.

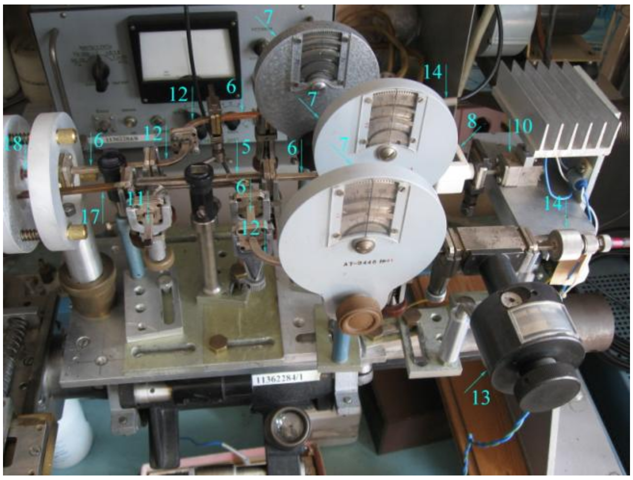

Figure 4: Appearance of the experimental stand

As EHF generator 10, a Gunn diode 3A728V (Fig. 3) operating at the second harmonic of the fundamental frequency is used. The generator frequency is 74.980 GHz, the output power is 35 mW. To expand the dynamic range p-i-n modulator 8 is included in the circuit. With its help, the EHF oscillations generated by the Gunn diode are modulated in an amplitude with a frequency of 1 kHz from sound generator 9. To decouple generator 10 and the resonator formed by mirrors 1 and 2, attenuator 7 is included in the circuit, its direct loss at the generator frequency is -8.5 dB. The circuit of the experimental stand includes directional coupler 5 connected to decoupling attenuator 7 through a waveguide 6 segment. An additional path is provided in the circuit to control the frequency of generator 10 and, if necessary, measure the reflection coefficient from the OR. It is composed of a segment of waveguide 6, 90º bend 12 in the *H*-plane of the TE_{10} main wave of a rectangular waveguide, measuring polarization attenuator 7, resonant wavemeter 13, detector 14, selective amplifier 15 and oscilloscope 16 (Fig. 3, 4). Matched load 11 is included in the path to avoid the impact of some power of generator 10 branched off in the forward direction of coupler 5 on the measurement results.

The resonator is excited by slot coupling element 17, which is a smooth transition from a reduced section of 3.6×0.15 mm to the main section of a rectangular waveguide 3.6×1.8 mm (Fig. 3, 6). The distance from the axis of spherical mirror 2 to the center of slotted coupling element 17 is 11.5 mm. It is determined by the maximum value of the electric field strength of the highest TEM_{3012} mode ($L/R \approx 0.65$) on spherical mirror 2 of the resonator. The mode is assumed to have the maximum value of the loaded q-factor Q_L. In this case it refers to the second maximum of the electric field strength when counting from the axis of the resonator (Fig. 1). To determine the location of coupling element 17 the expression (1) should be differentiated with respect to *p* and the resulting equation is equated to zero. As a result of its solution $\rho_4 = 1.438 w_1$. Here, w_1 is the spot radius of the field of the main TEM_{0012} mode on the spherical mirror of the resonator. For this mode and the given frequency of generator 10, the estimated value of $L/R \approx 0.6$ [16]. For values λ = 4.001 mm and *R*=41 mm from the formula [17]

$$w_1 = \sqrt{\lambda R/\pi \sqrt{L/R/(1-L/R)}}\,, \qquad (16)$$

w_1 =7.997 mm is found. Hence $\rho_4 = 11.5$ mm is obtained.

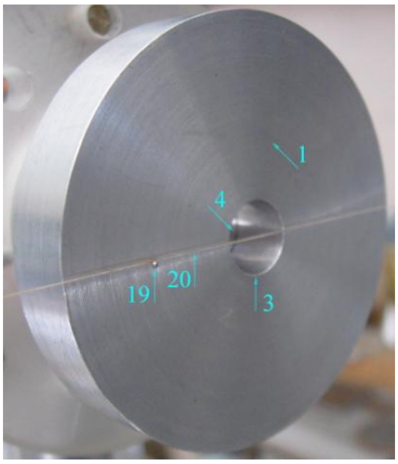

Figure 5: Flat mirror OR with a segment of a circular waveguide

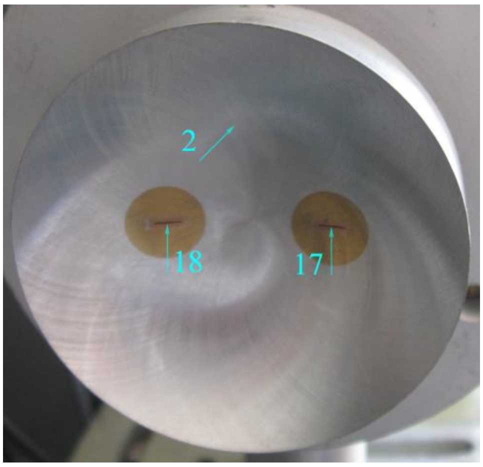

Figure 6: Spherical mirror OR with slotted coupling elements

By analogy, the radius of the field spot w_0 of the main TEM_{0012} mode on the flat mirror of the resonator is calculated using the formula [17].

$$w_0 = \sqrt{\lambda R/\pi \sqrt{(L/R)(1-L/R)}}\,. \qquad (17)$$

For the indicated values of λ, *R* and *L/R* w_0=5.058 mm. It was shown in the previous section that the radius *a* of a circular waveguide segment made at the center of a flat mirror OR should be equal to $0.993 w_0$. In this case, the TE_{01} wave will be excited in the waveguide with maximum efficiency using the TEM_{30q} mode. For this case *a*=5.023 mm. Hence it becomes clear why the diameter of the round waveguide 3 (Fig. 3, 6) equal to 10 mm has been chosen. Therefore, it can be said that this waveguide is oversized, since its diameter exceeds two wavelengths. This confirms the correctness of the above assumption about the absence of reflection from the open end of the circular waveguide. It should be noted here that the resulting diameter of a circular waveguide is much smaller than the optimal diameter of the same waveguide (18 mm) when the axially asymmetric TEM_{01q} mode is excited in the resonator [7]. This is precisely due to the high transverse index of the TEM_{30q} mode.

The signal from the resonator is output using the second slotted coupling element 18 (Fig. 3, 6). It also represents a smooth transition from a reduced section of 3.6×0.15 mm to the main section of a rectangular waveguide 3.6×1.8 mm. Measuring polarization attenuator 7 is connected to the standard output of this waveguide through two segments of rectangular waveguides 6 and two 90º bends 12 in the *H*- plane of the TE_{10} main wave of the rectangular waveguide. The signal is fixed by detector 14 at the attenuator 7 output and then it is delivered to selective amplifier 15 and oscilloscope 16 (Fig. 3, 4). Coupling element 18 is located on the resonator spherical mirror diametrically opposite to coupling element 17 (Fig. 6). Its distance from the resonator axis is also 11.5 mm.

The measurement procedure is as follows. We move spherical mirror 2 OR and fix the resonance by the maximum signal on the oscilloscope 16 screen included in the path of slotted coupling element 18. Now, using test body 19 fixed on 0.1 mm thick nylon thread 20 (Fig. 3, 5), it is determined that there is exactly the TEM_{30q} mode in the resonator [17]. In order to find the resonant transmission coefficient K_{transm}, for each specific distance between the OR mirrors, the procedure that is described in detail in [18] is made use of.

IV. RESULTS OF EXPERIMENTAL STUDIES

At the first stage, the piston is flush to the flat mirror OR. The highest axially asymmetric TEM_{30q} mode is excited in the resonator. Consider the behavior of the resonant transmission coefficient K_{transm} OR as the distance between the mirrors *L/R* decreases. The measurement results are shown in fig. 7 (curve 1). As the distance between the resonator mirrors decreases, the diffraction losses decline. This corresponds to a sharp increase in K_{transm}. Starting from the value $L/R=0.646$ ($K_{transm}=0.394$), corresponding to TEM_{3012} mode, the resonant transmission coefficient changes slightly. This K_{transm} behavior at $L/R<0.646$ indicates that the losses in the resonator are determined by the ohmic losses in the mirrors. The losses in the resonator are determined mainly by diffraction losses at $L/R>0.646$. With regard to the TEM_{3012} mode, both types of losses are approximately equal for the specified distance between the mirrors. This distance corresponds to the maximum value regime of its own *Q*-factor Q_0. To find the loaded *Q*-factor Q_L of the TEM_{3012} mode, the formula $Q_L=L/\Delta l$ is used [19]. Here *L* is the distance corresponding to the maximum value of the resonant transmission coefficient, $\Delta l=L_1-L_2$. The values L_1 and L_2 correspond to the distances between the resonator mirrors at which the transmission coefficient decreases by -3dB. This approach is due to the fact that work is done at a fixed frequency. As a result of the measurements, it has been shown that $Q_L=2960$ for the TEM_{3012} mode.

The figure demonstrates a sharp drop of K_{transm} to a value of 0.253 at $L/R=0.49$. This distance between the OR mirrors corresponds to the semi-confocal geometry of the resonator at which the oscillations always degenerate. In that case the TEM_{309} mode interacts with the TEM_{109} mode. Both modes have the same symmetry class. This result is consistent with the data of [18].

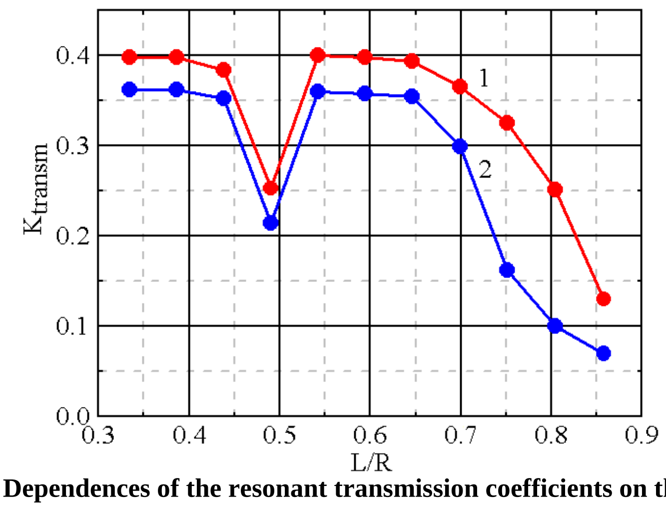

Figure 7: Dependences of the resonant transmission coefficients on the distance between the OR mirrors upon excitation of the TEM_{30q} mode

Now, at the moment when the TEM_{3012} mode exists in the resonator, piston 4 in the round waveguide is moved by means of a micrometric screw until the maximum signal is obtained on the screen of oscilloscope 16 included in the path of slotted coupling element 18 (Fig. 4). In this case, the length of the circular waveguide segment was 9.192 mm. It is well known that the wavelength in a waveguide λ_w is given by [20]

$$\lambda_w = \lambda \Big/ \sqrt{1-(\lambda/\lambda_{crit})^2} \,. \tag{18}$$

Here λ_{crit} is the critical wavelength of TE_{01} wave in a circular waveguide equal to $1.64a$. After substituting the values $\lambda=4.001$ mm and $a=5$ mm into expression (18), $\lambda_w=4.584$ mm is obtained. The two waveguide wavelengths are 9.168 mm. Thus, the length of a circular waveguide segment with a diameter of 10 mm is $2\lambda_w$. The relative error of the measured length of a circular waveguide segment with respect to the calculated length is 0.153%. This is an indirect evidence of the TE_{01} wave propagation in the waveguide, besides the fact that the piston does not touch the walls of the circular waveguide.

Without changing the position of the piston, consider the behavior of K_{transm} when changing *L/R*. It is assumed that a degenerate axially symmetric TEM^{*}_{11q} mode [10] is excited in the resonator in this case. This assumption is due to the fact that, as shown in [14], the presence of a round oversized waveguide at the center of one of the mirrors leads to the resonator mode transformation into a degenerate axially symmetric one. A distinctive feature of such modes in OR is that they are characterized only by the electric field component $\vec{E}_{\varphi}$. The outer ring of the electric field strength of the TEM^{*}_{11q} mode will be located on the surface of the flat mirror. The inner ring of the electric field strength of this mode should excite the TE_{01} wave in a circular waveguide with high efficiency (Fig. 5).

The measurement results are shown in fig. 7 (curve 2). In this case, the overall behavior of the resonant transition coefficient with changing *L/R* is preserved. The above figure shows that, as in the previous case, when the distance between the resonator mirrors decreases, the diffraction losses go down. Hence, there is a sharp increase in K_{transm}. Starting with *L/R*<0.646, the losses in the resonator are determined mainly by the ohmic losses in the OR mirrors. This leads to the fact that the resonant transmission coefficient changes slightly with decreasing *L/R*. In this case, the TEM^{*}_{1112} mode will have the maximum value of its own *Q*-factor Q_0 (K_{transm}=0.354), for which *L/R*=0.646. Both types of losses are equal for it. Estimate the value Q_L of the TEM^{*}_{1112} mode by analogy with what was done above. The loaded *Q*-factor of this mode is 2850.

It can be seen from the figure that at *L/R*=0.49 K_{transm} drops sharply to a value of 0.214. This distance between the OR mirrors, as mentioned above, corresponds to the semi-confocal geometry of the resonator. In this case, the TEM^{*}_{119} and TEM^{*}_{109} modes interact [14].

Loaded *Q*-factor is a measure of losses in the resonator. The studies performed show that the presence of a circular waveguide segment at the center of a flat mirror OR leads to a decrease of Q_L by 10% for the mode under consideration. In absolute terms, the presence of a circular waveguide segment leads to an increase in losses in the resonator by 0.9 dB for the TEM^{*}_{1112} mode compared to the TEM_{3012} mode. This indicates that the TE_{01} wave in the circular waveguide segment is excited with high efficiency. Thus, the losses in the OR increase insignificantly with the presence of an oversized circular waveguide segment. This result is of great importance when such a resonant system is used as a storage resonator for an electromagnetic pulse compressor in the EHF range.

Analyze the field structure of the TEM_{3012} and TEM^{*}_{1112} modes in a hemispherical OR. To do this, the trial body method is used [17]. The electric component distribution of the standing wave field in the resonator will be measured using a test body 19 fixed on a nylon thread 20 (Fig. 3, 5). The test body is a scattering metal ball. To select the diameter of the test body *d*, the formula [21] is used

$$d = \lambda 10^{0.062 - \frac{\lg Q_L}{4.938}}. \tag{19}$$

Taking into account the values of the loaded *Q*-factors of the considered modes and λ=4.001 mm from expression (19), the diameter of the test body is found as *d*=0.9 mm. Piston 4 in round oversized waveguide 3 is located in the plane of mirror 2 (Fig. 4, 5). In this case, the TEM_{3012} mode is excited in the resonator at *L/R*=0.646. The electric field distribution will be measured in the plane of the $\vec{H}$ vector of the TE_{10} wave in slotted coupling element 17 made on a spherical mirror (Fig. 6). The measurements are carried out in the first antinode of the electric component of the standing wave field in the resonator, counting from a flat mirror. The measurement results are shown in fig. 8 (curve 1). This is the distribution of the electrical component field of the TEM_{3012} mode in the plane coinciding with the plane of Fig. 1. For greater clarity, move on to the Cartesian coordinate system when plotting graphs.

In order to build the calculated distribution of the TEM_{3012} mode field on the flat mirror of the resonator, the expression (1) is used, in which we set $\rho = x$, $\varphi = 0$ (*y*=0). Above, the distance from the resonator axis to the second maximum of the electric field strength on the spherical mirror was found to be equal to $\rho_4 = 1.438 w_1$ for the TEM_{30q} mode. Now consider the same mode on a flat resonator mirror. The distance to the second maximum of the electric field strength on the flat mirror will be equal to $\rho'_4 = 1.438 w_0$. Here w_0 is the spot radius of the TEM_{00q} main mode field on the flat mirror OR. After substituting the value ρ'_4 into expression (1) we obtain $E_e^{\max}(x) = 5.423 E_0$. Then the expression for the normalized distribution of the electric field of the TEM_{3012} mode on the flat mirror OR takes the form

$$\left(E_e(x)/E_e^{\max}(x)\right)=4\left(\sqrt{2}x/w_0\right)\exp\left(-x^2/w_0^2\right)\left[2\left(\sqrt{2}x/w_0\right)^2-3\right]/5.423. \quad (20)$$

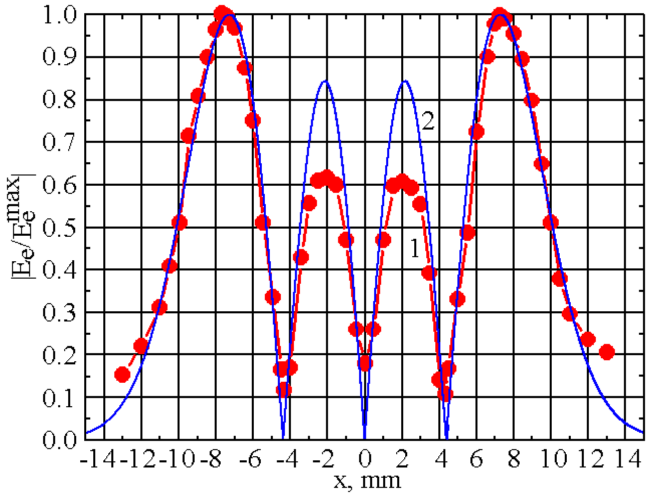

Figure 8: Distribution of the electric field strength of the TEM_{3012} mode in a hemispherical OR

For the TEM_{3012} mode, we got above that the w_0=5.058 mm. The calculation results by formula (20) are presented in Fig. 8 (curve 2). The above figure demonstrates a good agreement between the measured and calculated distributions of the electric component field of the TEM_{3012} mode in the OR. This concerns the external spots of the fields of the modes under consideration. The measured and calculated maximum values of the electric field strengths of the internal spot fields of the TEM_{3012} mode differ significantly. The measured electric field strength of the mode under consideration falls off on both sides of the resonator axis in a plane perpendicular to the plane of Fig. 1 (plane $y0z$). In this case, $E_e(x)/E_e^{\max}(x)$=0.178 on the resonator axis. The difference from zero of the electric field strength of the TEM_{3012} mode, where the calculated value $E_e(x)/E_e^{\max}(x)$ should be equal to zero, is associated with the finite dimensions of the test body.

Now move the piston to a distance of 9.182 mm from the mirror surface. In this case, there must be a degenerate axially symmetric TEM_{1112}^* mode in the resonator (L/R=0.646). Fig. 9. (curve 1) shows the measured distribution of the electric field strength of this mode in the plane of Fig. 1.

It can be seen from the figure that the presence of an oversized circular waveguide segment in the center of the flat mirror OR leads to the fact that the outer diameter of the inner ring of the TEM_{1112}^* mode becomes larger than the calculated one (curve 2) made according to formula (20). It actually coincides with the diameter of a circular waveguide segment, which is 10 mm. The maximum of the electric field strength in this ring also shifts from the resonator axis towards larger values (~2.5 mm). The calculated value of this maximum is $\rho_2=0.426w_0=2.154$ mm. Thus, the field structure in this ring repeats the electric field structure of the TE_{01} wave in a circular waveguide segment. An increase in the diameter of the inner ring of the TEM_{1112}^* mode also leads to the shift of the outer ring of the mode under consideration in the direction of the mirror periphery. As in the previous case, the measured and calculated maximum values of the electric field strengths of the inner ring of the considered mode differ.

It is of practical interest to evaluate the behavior of the electric field strength of the TEM_{1112}^* mode in a plane perpendicular to the plane of Fig. 1 (plane $y0z$). The measurement results are shown in fig. 10 (curve 1). It also shows the calculated curve, made according to formula (20). It can be seen here, that, as in the previous case, the measured and calculated maximum values of the electric field strengths of the inner ring of the TEM_{1112}^* mode differ significantly. In addition, the outer diameter of the inner ring of the mode under consideration has increased up to the diameter of a circular waveguide. The maximum electric field strength in this ring practically coincides with the maximum value of the electric field strength of the TE_{01} wave in a circular waveguide. By increasing the diameter of the inner ring, the outer diameter of this mode on the resonator mirror rises. The experimental studies carried out have showed that the oscillation of the resonator becomes axially symmetric in the presence of an oversized circular waveguide segment at the center of a flat mirror OR. This confirms the correctness of the above assumptions. When the TE_{01} wave is excited in a circular waveguide, zero values of the electric field strength fall on the edge of the circular waveguide. Thus, the waveguide wave modifies the mode of the resonator.

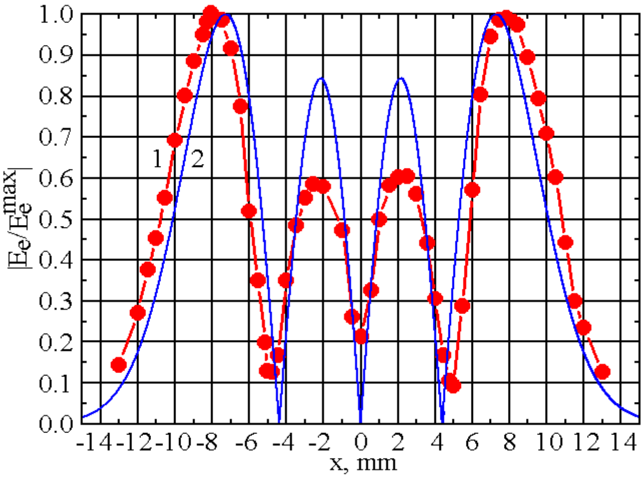

Figure 9: Distribution of the electric field strength of the TEM^*_{1112} mode in a hemispherical OR in the *x*0*z* plane

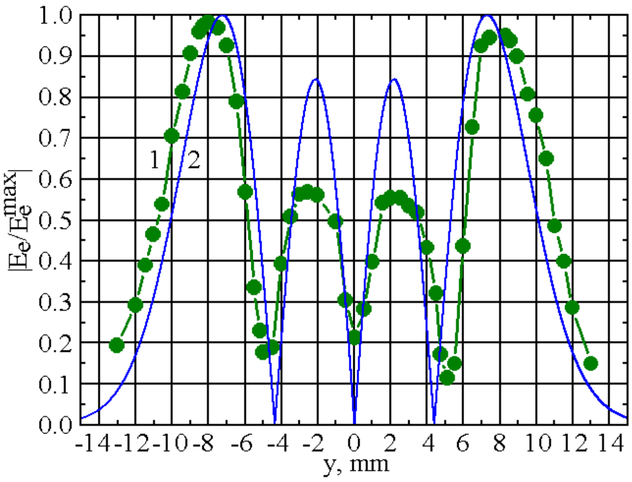

Figure 10: Distribution of electric field strength of TEM^*_{1112} mode in a hemispherical OR in the *y*0*z* plane

In order to demonstrate more clearly what a degenerate axially symmetric TEM^*_{1112} mode is, expression (1) and the value w_0=5.058 mm will be used. The results of the calculation are shown in Fig. 11. When the given mode is compared with the similar mode described in [10], it can be seen easily that they differ. The amplitude in the central ring is always higher than in the outer one in the well-known degenerate axially symmetric TEM^*_{11q} mode. Similar modes are described by Laguerre-Gaussian functions. In the mode under the consideration, the reverse is true.

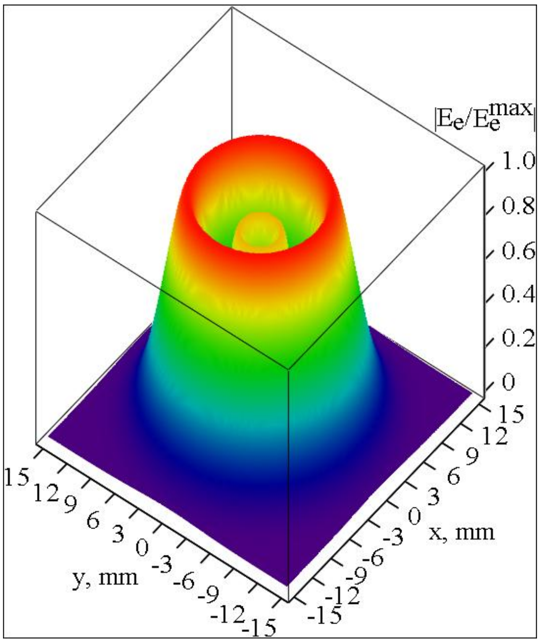

Figure 11: Degenerate axially symmetric TEM^{*}_{1112} mode

The mode of this type is described by Hermite-Gauss functions. Thus, using an oversized circular waveguide segment, we have succeeded in exciting a degenerate axially symmetric mode in the OR, which does not exist in nature without an additional element in the resonator (a segment of a circular waveguide).

V. CONCLUSIONS

The studies carried out in this work allow to draw a number of important practical conclusions.

1. The presence of a round oversized waveguide segment, made in the center of one of the resonator mirrors, leads to the fact that the axially asymmetric TEM_{30q} mode is converted into a degenerate axially symmetric TEM^{*}_{11q} mode.

2. The excitation efficiency of the TE_{01} wave in a circular waveguide with the help of higher TEM_{30q} and TEM^{*}_{11q} modes OR is not high. This wave is excited in the waveguide with a maximum efficiency of 0.955 using the inner field ring of the degenerate axially symmetric TEM^{*}_{11q} mode. In this case, the radius of a round oversized waveguide a, made in the center of one of the OR mirrors, should be equal to $0.993w_0$. Here w_0 is the spot radius of the field of the main TEM_{00q} mode on this mirror.

3. With the optimal choice of the diameter of the circular waveguide, the losses associated with the transformation of the inner ring of the TEM^{*}_{11q} mode field into the TE_{01} waveguide wave will be minimal. The experimental studies carried out have shown that the losses in the resonator increased by only 0.9 dB in this case.

4. The TE_{01} wave of a circular waveguide and the TEM^{*}_{11q} mode of the OR are interconnected. This leads to the fact that the inner ring of the field spot of the TEM^{*}_{11q} mode becomes similar to the TE_{01} wave in its structure.

5. The OR considered in the work can be used as a storage resonator in the construction of electromagnetic pulse compressors in the EHF range. This is due to the fact that the losses in the resonant system increase insignificantly, and the round waveguide itself is oversized. Due to this fact, it is possible to output the power accumulated in the resonant volume to the load over a short period of time using an interference key.

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