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Determination of Optimum Working Interest Using Hyperbolic Utility-Type Inversion; an Oil and Gas Asset Valuation Approach

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ABSTRACT
Not all decision makers model their risk attitudes and behavior with cozzolino's exponential utility model, some employ empirical models based on the historical evaluations of similar prospect and its expected values, while a few others use the hyperbolic tangent type of risk weighing. This may be due fact that very few researchers have full understanding of how the hyperbolic utility model works and how it was derived. However, the hyperbolic model, unlike the exponential has very few published works available for research. This project presents a detailed study on the hyperbolic tangential utility function for modeling risk. This work illustrates a new and different approach in evaluating the risk-adjusted value (RAV), apparent risk tolerance, and how optimum working interest can be derived mathematically other than the approximate solutions given by Lerche 1999.
KEYWORDS— *Exponential , Hyperbolic, Optimum Working Interest, Risk Adjusted Value and Investment.*

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I. INTRODUCTION

The oil and gas business is an inherently risky business that is capital intensive and also possesses a whole lot of uncertainties associated with the business, due to the large amount resources ranging to billions of dollars. Decision makers are regularly confronted with the challenge of evaluating their assets, quantifying risk, allocating the available resources among a set of available projects - projects generally characterized by a high degree of financial risk and uncertainty.

One of the more interesting hydrocarbon economic challenges encountered by decision makers over the last decade or so has been associated with estimating risk adjusted value RAV at optimum and breakeven working interests a venture should take in an opportunity under conditions of high risk. However, it has long been recognized that the Exponential Model is only one of a class of Utility functions among many others that is widely used to model risk attitudes. Others make use of empirical models based on prior project evaluations. Some Corporations use the hyperbolic tangent form of risk aversion and estimation of risk adjusted value and working interest.

Problem Statement

The law of gambler's ruin states that there is always a probability of going bankrupt by a normal run of a series of bad luck irrespective of the long run expectations. This can be avoided by a strategy commonly employed in the Oil and Gas Industry called risk sharing or simply taking a fractional part of a prospect. A decision maker's selection of the desired working interest (level of participation) establishes its fundamental risk aversion. However, considering the fact that the oil and gas business is capital intensive, decision makers are usually careful in selecting its working interest so as to minimize losses and maximize potential gains(Cozzolina, 1977). In modeling one's risk adjusted value RAV and working interest one of the several methods used is the hyperbolic tangent utility function. Lerche and Mackay, 1999 did develop a method along the line of Cozzolino's exponential model using the Hyperbolic tangent utility function. However, Lerche and Mackay's model was not as explicit as Cozzolino's exponential model, hence its limited usage. They arrived at an implicit model for Working Interest determination using the hyperbolic tangent function to model risk and exponential inversion to translate utilities to monetary values. So their model can be termed a hybrid model. This work investigates the determination of working interest using the hyperbolic tangent model and hyperbolic type inversion scheme.

II. LITERATURES

A practical alternative to determine venture participation when the decision maker's level of risk aversion changes with profitability and risk investment (McKay, 1975) ; in this method, decision makers evaluate their fundamental risk aversion in all ventures when the expected profit equals the risk investment in each venture. A decision maker's selection of a desired participation level for any venture that anticipates only a discounted payout establishes his fundamental risk aversion. Venture participation is determined after a thorough review of the interrelation of venture profitability, total risk investment, risk aversion S, probability of success P_s, and available risk investment funds M (Greenwolt, 1981)

Given that the probability of failure is equal to probability of dry holes (Mian, 2002). He derives the following expression for determining venture participation F

$$\boldsymbol{F} = \frac{\boldsymbol{H}\log(\mathbf{1}-\boldsymbol{P_s})}{\log(\mathbf{1}-\boldsymbol{R})} \qquad \text{1}$$

Where H is the desired working interest, Risk capacity *(R)* is the ratio of one successful venture to the one success and the maximum number of unsuccessful venture expenditures recovered by the profit of that success.

Equation 1 is used to determine venture participation without having quantifying risk aversion in terms of S (Quick, 1984). Decision makers determine their fundamental aversion to risk in this equation by the selection of *H* for any venture that anticipates only a discounted payout. Decision makers who choose to evaluate his venture participation using this method must first establish the appropriate value of *H* for the venture.

EXPECTED VALUE (EV) APPROACH

Decision making theory proposes that when choosing between alternative projects, the preferred or most likely selected project is one which maximizes value. This approach explicitly makes use of the probability of occurrence of each of the various outcomes of an investment. The Expected Value is in fact, the probability weighted average or the mean of the various value outcomes of an investment. It includes the probability of occurrence of all the events in a prospect(Baird, 1989).

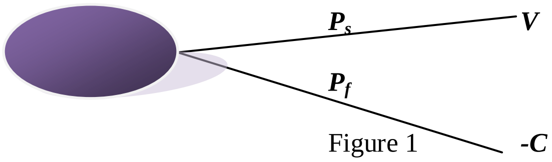

Figure 1

This is a two outcome prospect such as in wild cat drilling in which the outcomes are Discovery with present value, V or Dry hole, with a loss of C, the cost of the drilling operation and any other upfront Exploration costs.

DETERMINATION OF OPTIMUM WORKING INTEREST

The risk adjusted value RAV is a non linear function of working interest (W) – thus there is an optimum working Interest at the risk adjusted value is maximum. Differentiating with respect to working interest, one can derive the optimum working interest that maximizes risk adjusted values RAV(Chavasjean, 2004). The formula for optimum working interest can be derived by differentiating the risk adjusted values RAV with respect to working interest and setting the derivative to zero(Rosenberg,1985)

$$\boldsymbol{RAV} = \boldsymbol{RT}\tanh^{-1}\left[\boldsymbol{P_s tanh}\left(\frac{\boldsymbol{WV}}{\boldsymbol{RT}}\right) - \boldsymbol{P_f tanh}\left(\frac{\boldsymbol{WC}}{\boldsymbol{RT}}\right)\right]$$

Differentiating **RAV** and setting the derivative $\frac{dRAV}{dw} = \mathbf{0}$ and simplifying the equation yield

$$\cosh\frac{W_{opt}C}{RT} = \left(\frac{P_f C}{P_S V}\right)^{\frac{1}{2}}\cosh\left(\frac{W_{opt}V}{RT}\right) \qquad \textbf{2}$$

Using Laplace Transformation

Recall,

$$L\{F(t)\} = \int_{t=0}^{t=\infty} f(t)e^{-st}\,dt = F(s) \qquad 3$$

$$\mathrm{F}(t) = \cosh(at)$$

$$\boldsymbol{L\{F(t)\}} = \int_{t=0}^{t=\infty}(\cosh(at))e^{-st}\,dt = \frac{s}{s^2-a^2} \qquad \textbf{4}$$

Therefore, simplifying the Laplace transforms of equation 2 yield solving for S

$$\frac{1}{s^2} = \frac{(P_sV)^{\frac{1}{2}} - (P_fC)^{\frac{1}{2}}}{(P_sV)^{\frac{1}{2}}(\frac{V}{RT})^2 - (P_fC)^{1/2}(\frac{C}{RT})^2} \ldots\ldots 5$$

Using Laplace Inverse Transformation

$$L^{-1}\left\{\frac{1}{s^2}\right\} = t = W_{opt} \ldots\ldots 6$$

Therefore, the inverse transforms of equation 5 yield

$$W_{opt} = \frac{(P_sV)^{\frac{1}{2}} - (P_fC)^{\frac{1}{2}}}{(P_sV)^{\frac{1}{2}}\left(\frac{V}{RT}\right)^2 - (P_fC)^{\frac{1}{2}}\left(\frac{C}{RT}\right)^2} \ldots\ldots 7$$

Therefore, optimum working interest (W_{opt}) using hyperbolic tangent utility-type inversion

$$W_{opt} = \frac{RT^2\left((P_sV)^{\frac{1}{2}} - (P_fC)^{\frac{1}{2}}\right)}{(P_sV)^{\frac{1}{2}}(V)^2 - (P_fC)^{\frac{1}{2}}(C)^2}$$

LERCHE & MACKAY (HYPERBOLIC UTILITY) APPROACH

Risk Adjusted Value RAV Analysis

Use of Hyperbolic Utility Function form:

$$U(x) = 1 - tanh(x) \ldots\ldots *1$$

Where x = terminal wealth and r = risk aversion level =1/millionths

The Expected Utility (EU) of the prospect in Figure 2.1(Lerche, 1993) can be expressed by the following:

$$EU(x) = P_s\left[1 - \tanh\left(\frac{WV}{RT}\right)\right] + P_f\left[1 - tanh\left(\frac{-WC}{RT}\right)\right] \ldots\ldots *3$$

$$EU(x) = \left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \ldots\ldots *4$$

The Certainty Equivalent (CE) of this expected utility is the Risk Adjusted value and Lerche & Mackay assumed it is also of the *exponential form* and expressed it as:

$$e^{-\frac{RAV}{RT}} = \left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \ldots\ldots *5$$

$$RAV = -RT\, ln\left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \ldots\ldots *6$$

The equation is equivalent to Cozzolino's formula, but for hyperbolic tangent weighting of risk aversion rather than exponential (Smith, 2003)

Optimum Working Interest

The Risk Adjusted Value RAV) is also a non linear function of the Working Interest, WI. Differentiating the risk adjusted value RAV equations *6 with respect to WI and equating the derivative to zero(Capen 1976), RAV has maximum value at Working Interest expressed implicitly by:

$$cosh\left(\frac{W_{opt}C}{RT}\right) = \left(\frac{P_fC}{P_sV}\right)^{1/2} cosh\left(\frac{W_{opt}V}{RT}\right) \ldots\ldots *7$$

Solving the explicit equation yield an approximate solution of

$$WI_{opt} \cong \frac{RT}{2V}\ln\left(\frac{4P_sV}{P_fC}\right) \ldots \ldots * 8$$

Aim of study
The aim of this work is formulating mathematically a model for estimating risk adjusted value RAV and working interest at optimum and at breakeven using the Hyperbolic tangent utility functions and hyperbolic type inversion scheme.

Objectives of this study
To derive an empirical formula for estimating risk adjusted value (RAV) and optimum working interest using the hyperbolic tangent utility functions and hyperbolic type inversion scheme and to study the effect of changes in net present value (NPV), cost of investment, and success chance factor on the optimum working interest.

Justification of study
Lerche & Mackay, in their investigation of the use of the hyperbolic model used the hyperbolic function to model risk but the exponential form for inversion into real monetary values making its model a hybrid model. However, this model uses hyperbolic function to model risk and also inversion into real monetary values.

Scope of study
This study is limited to evaluation of risk adjusted value, optimum working interest and breakeven working interest by hyperbolic tangent utility function.

III. METHODOLOGY

Model development simply involves the mathematical analysis of the hyperbolic utility function and the direct substitution of the hyperbolic tangent of the utility function as the certainty equivalent (C.E) unlike the Lerche hyperbolic model that uses an exponential certainty equivalent (C.E) and a further resolving of risk adjusted value using Laplace transform and Inverse Laplace Transform to solve for an exact solution of the optimum working interest and the breakeven working interest. Data from C.k Moore (2005) and Lerche is utilized to test the model.

DETERMINATION OF RISK ADJUSTED VALUE RAV
Given the Utility function hyperbolic form:

$$U(x) = 1 - tanh(xr) \ldots\ldots 8$$

$$r = {}^{1}/_{RT}$$

U(x) = Tanh (x/RT)
where x represents wealth and RT is the risk tolerance in monetary values
The Expected Utility (EU) of the prospect can be expressed by the following:

$$EU(x) = \left[P_s tanh\left(\frac{WV}{RT}\right) - P_f tanh\left(\frac{WC}{RT}\right)\right] \ldots\ldots 9$$

The Certainty Equivalent (CE) of this expected utility is the Risk Adjusted value when expressed in hyperbolic form is given by

$$\tanh\frac{RAV}{RT} = \left[P_s tanh\left(\frac{WV}{RT}\right) - P_f tanh\left(\frac{WC}{RT}\right)\right]$$

$$RAV = RT\tanh^{-1}\left[P_s tanh\left(\frac{WV}{RT}\right) - P_f tanh\left(\frac{WC}{RT}\right)\right] \ldots\ldots 10$$

This gives the formula for evaluating risk adjusted value RAV using hyperbolic tangent utility function.

DETERMINATION OF OPTIMUM WORKING INTEREST

The risk adjusted value RAV is a non linear function of working interest (W) – thus there is an optimum working Interest at the risk adjusted value is maximum. Differentiating with respect to working interest, one can derive the optimum working interest that maximizes risk adjusted values RAV. The formula for optimum working interest can be derived by differentiating the risk adjusted values RAV with respect to working interest and setting the derivative to zero.
Recall, the RAV equation 10

$$\boldsymbol{RAV = RT\tanh^{-1}\left[P_s tanh\left(\frac{WV}{RT}\right) - P_f tanh\left(\frac{WC}{RT}\right)\right]}$$

Differentiating **RAV** and setting the derivative $\frac{dRAV}{dw} = 0$ and simplifying the equation yield

$$\boldsymbol{\cosh\frac{W_{opt}C}{RT} = \left(\frac{P_fC}{P_sV}\right)^{\frac{1}{2}}\cosh\left(\frac{W_{opt}V}{RT}\right) \ldots\ldots 11}$$

Using Laplace Transformation
Recall,

$L\{F(t)\} = \int_{t=0}^{t=\infty} f(t)e^{-st}\, dt = F(s)$……………………………..……………………12

$\mathrm{F}(t) = \cosh(at)$

$\boldsymbol{L\{F(t)\} = \int_{t=0}^{t=\infty}(\cosh(at))e^{-st}\, dt = \frac{s}{s^2-a^2}}$**……………………………………………….…....13**

Therefore, simplifying the Laplace transforms of equation 11 yield solving for S

$$\frac{1}{s^2} = \frac{(P_sV)^{\frac{1}{2}} - (P_fC)^{\frac{1}{2}}}{(P_sV)^{\frac{1}{2}}(\frac{V}{RT})^2 - (P_fC)^{1/2}(\frac{C}{RT})^2} \quad \text{……………………………………………….………………14}$$

Using Laplace Inverse Transformation

$L^{-1}\left\{\frac{1}{s^2}\right\} = t = W_{opt}$……………………………………………………………..……………15

Therefore, the inverse transforms of equation 14 yield

$$W_{opt} = \frac{(P_sV)^{\frac{1}{2}} - (P_fC)^{\frac{1}{2}}}{(P_sV)^{\frac{1}{2}}\left(\frac{V}{RT}\right)^2 - (P_fC)^{\frac{1}{2}}\left(\frac{C}{RT}\right)^2} \quad \text{……………………………………………………………………16}$$

Therefore, optimum working interest (W_{opt}) using hyperbolic tangent utility-type inversion

$$W_{opt} = \frac{RT^2\left((P_sV)^{\frac{1}{2}} - (P_fC)^{\frac{1}{2}}\right)}{(P_sV)^{\frac{1}{2}}(V)^2 - (P_fC)^{\frac{1}{2}}(C)^2}$$

RISK ADJSTED VALUE RAV AT OPTIMUM WORKING INTEREST

The RAV at optimum can be evaluated by substituting the optimum working in equation 16 into the RAV equation of 10

Recall;

$$RAV_{opt} = RT\tanh^{-1}\left[P_s\tanh\left(\frac{W_{opt}V}{RT}\right) - P_f\tanh\left(\frac{W_{opt}C}{RT}\right)\right]$$

Substituting W_{opt} of equation into equation 3.3 yield

$$RAV_{opt} = RT\tanh^{-1}\left[P_s\tanh\left(\left(\frac{RT\left((P_sV)^{\frac{1}{2}} - (P_fC)^{\frac{1}{2}}\right)}{(P_sV)^{\frac{1}{2}}(V)^2 - (P_fC)^{\frac{1}{2}}(C)^2}\right)V\right) - P_f\tanh\left(\left(\frac{RT\left((P_sV)^{\frac{1}{2}} - (P_fC)^{\frac{1}{2}}\right)}{(P_sV)^{\frac{1}{2}}(V)^2 - (P_fC)^{\frac{1}{2}}(C)^2}\right)C\right)\right] \text{………}$$

DISCUSSION: A NUMERICAL ILLUSTRATION

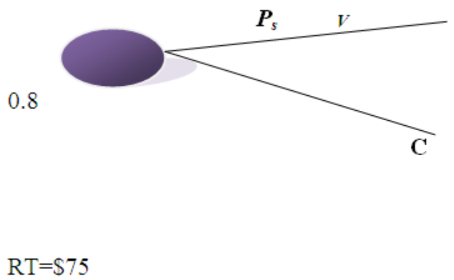

Figure 2a

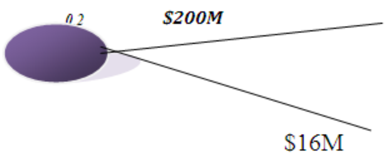

Figure 2b

The figure above illustrate a prospect as was used in Moore (2005), it indicates a projects with a Net present Value (V) of $200,000,000 and a cost (C) of $16,000,000 is required for executing the project, with the assumption that the chance of success (P_s) is 20%, probability of the project not succeeding (P_f) is 80% and assuming a risk tolerance (RT) of $75,000,000. The Expected Value (EV) of this project can be calculated as $27,200,000. It is expected that the decision maker would execute the project if the company has sufficient financial strength to mitigate in case of an unsuccessful outcome.

Assuming the company wishes to diversify its investment and only has about $16,000,000 and does not want to invest all of it i.e. 100% working interest in just this prospect and would wish to make funds available for other investment or desire to share the risk with another company interested in investing, the question is at what participation level (working interest) should each company undertake and what will be their breakeven working interest?

For model, the RAV and WI_{opt} of this prospect can be evaluated using eqn 10 and 16 respectively

$$\textit{Model}\ WI_{opt} = \frac{RT^2\left((P_sV)^{\frac{1}{2}} - (P_fC)^{\frac{1}{2}}\right)}{\left((P_sV)^{\frac{1}{2}}V^2 - (P_fC)^{\frac{1}{2}}C^2\right)} = 6.1\%$$

$$\textit{Model}\ RAV_{100\%} = RT\tanh^{-1}\left(P_s\tanh\left(\frac{WV}{RT}\right) - P_s\tanh\left(\frac{WC}{RT}\right)\right) = \$2247239$$

For Lerche and Mackey's model, RAV and WI_{opt} is evaluated using eqn *1 and *6 respectively

$$LercheWI_{opt} = \frac{RT}{2C}\ln\left(\frac{4P_sV}{P_fC}\right) = 47.4\%$$

$$Lerche\ RAV_{100\%\ WI} = -RT\ln\left(1 + P_f\tanh\left(\frac{WC}{RT}\right) - P_s\tanh\left(\frac{WV}{RT}\right)\right) = \$2280901$$

FURTHER NUMERICAL ILLUSTRATION FROM LITERATURE

Further examples from literature to investigate model's behaviour and to study the difference between models and Lerche.

1. Mackey 1995

	Prospect 1	Prospect 2	Prospect 3	Prospect 4	Prospect 5
P_s	0.2	0.25	0.15	0.8	0.5
V	500	2000	700	5	140
C	100	500	100	1	125.5
RT	1000	1000	1000	1000	1000
RAV_{LERCHE}	12.77022914	-100.371	5.955067	3.807205	7.149098
RAV_{MODEL}	12.68971685	- 105.976	5.937441	3.799985	7.123725
$WIopt_{LERCHE}$	69.90%	18.17%	49.56%	100%	100%
$WIopt_{MODEL}$	43.80%	3.54%	20.84%	100%	100%

Table 1

2. COZZOLINO 1977

	Prospect 1	Prospect 2	Prospect 3
P_s	0.2	0.2	0.2
V	0.81	0.81	0.403
C	0.19	0.1842	0.095
RT	0.128	0.193	0.257
RAV_{LERCHE}	-0.053748837	-0.06404	-0.0244
RAV_{MODEL}	-0.074092755	-0.08028	-0.02568
$WIopt_{LERCHE}$	4.98%	7.66%	20.01%
$WIopt_{MODEL}$	0.08%	0.28%	1.24%

Table 2

3. WALLS AND CO

	Prospect 1	Prospect 2	Prospect 3	Prospect 4	Prospect 5	Prospect 6	Prospect7
P_s	0.5	0.2	0.5	0.15	0.3	0.8	0.2
V	40MM	100MM	35MM	45	22	14	16
C	16MM	10MM	15MM	3	4	9.5	1.4
RT	25MM	25MM	25MM	25	25	25	25
RAV_{LERCHE}	4.912068897	-2.47561	4.783585	1.033775	2.658242	10.15565	1.766643

RAV_{MODEL}	4.507839865	-2.61179	4.398609	1.013247	2.530402	8.678605	1.708321
$WIopt_{LERCHE}$	31.25%	7.53%	34.64%	28.47%	55.37%	100%	82.66%
$WIopt_{MODEL}$	15.97%	2.31%	20.03%	11.93%	46.01%	100%	100%

Table 3

BREAKEVEN WORKING INTEREST

At break even risk adjusted value is 0 (RAV=0).

At breakeven, $RAV = 0$

$0 = RT\tanh^{-1}\left(P_s \tanh\left(\frac{WV}{RT}\right) - P_f \tanh\left(\frac{WC}{RT}\right)\right)$……………………………………………………1-1

Using Laplace transform

Given a function f(t)

$L\{f(t)\} = \int_{t=0}^{\infty} f(t)e^{-st}\,dt = F(S)$……………………………..……………………………1-2

Simplifying with $1 = P_s + P_f$ to get an equation for $\frac{1}{S^2}$, the Laplace transform of equation 1-1 yield

$\frac{1}{S^2} = \left(\frac{RT^2(2P_sV - P_fC)}{P_sV(V^2-C^2) - P_fC(C^2-V^2)}\right)$**……………………………………………………...……………1-3**

Using Laplace Inverse Transformation

$L^{-1}\left\{\frac{1}{S^2}\right\} = t = WI_{@breakeven}$…………………………………………………………………..….1-4

Therefore, the Breakeven working Interest (RAV=0) is given as

$WI_{@breakeven} = \frac{RT^2(2P_sV - P_fC)}{P_sV(V^2-C^2) - P_fC(C^2-V^2)}$………………..………………………………1-5

The breakeven working interest formulae above shows a form of resemblance to the optimum working interest which is expected. It is directly proportional to the square of risk tolerance RT same as model's Optimum working interest.

Below is a plot of breakeven working interest against risk tolerance

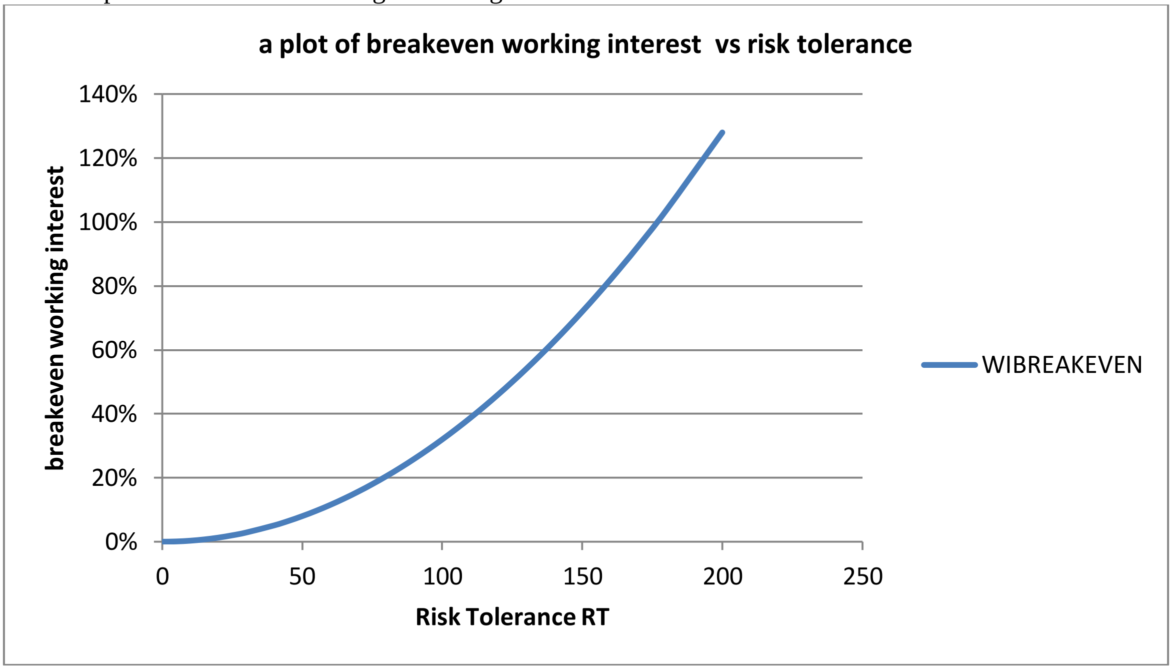

FIGURE 2 Breakeven working interest WI versus Risk Tolerance in \$MM).

Breakeven working interest has to constrained at 1 (100%).

Considering the parameters given in the numerical illustration of Figure 2b. A plot of the breakeven working interest against risk tolerance is made in figure 2 Again it can be noted that breakeven working interest increases with the risk tolerance this implies that the larger the risk tolerance value, the greater will be the breakeven working interest that should be taken in the project. The plot tends to curve initially at low risk tolerance value i.e. it is only proportional and linear at a much higher risk tolerance which is similar to the behavior exhibited by the optimum working interest.

Risk Adjusted Values RAV

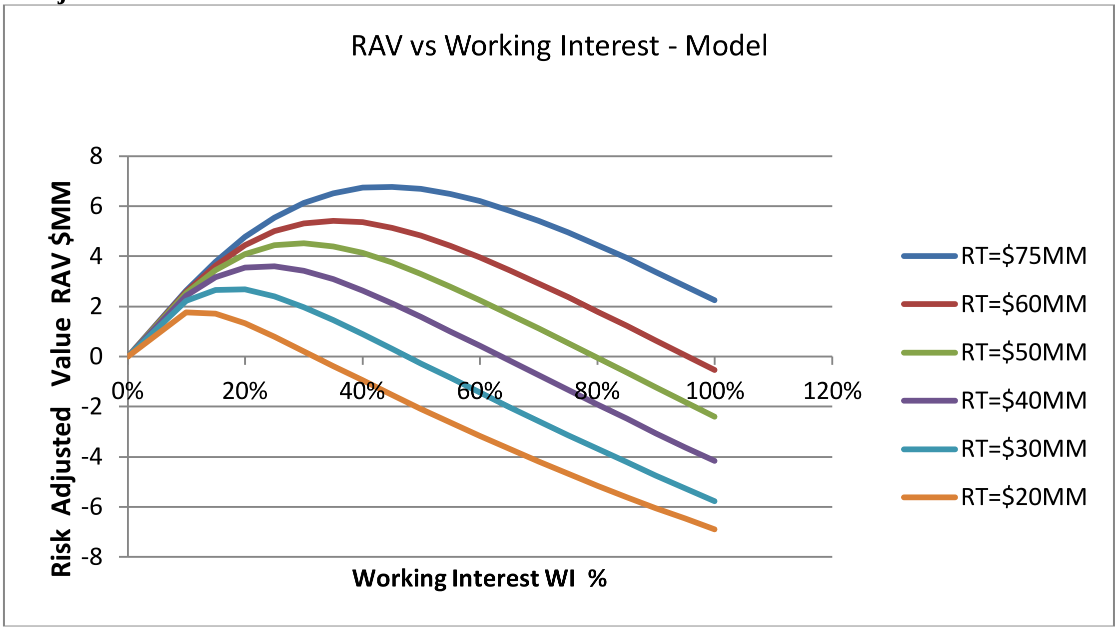

FIGURE 3 risk adjusted value *(RAV)* in $MM versus working interest, W(%), for various risk tolerance (labeled on each curve in $MM) for model.

Note that the *RAV* has positive and negative values until *RT* crosses about $65 MM, when any working interest up to 100% will be profitable. Model risk RAV formula was used with the data in the numerical illustration in figure 2b

With the parameters used in the illustration in Figure 2b, the plot of risk adjusted value (RAV) versus Working Interest for this portfolio at different risk tolerance RT is shown in figure 3. The risk adjusted values RAV increases with the working interest WI up to certain maximum value and start to decrease, the value of the working interest at which the risk adjusted value RAV is maximum is the Optimum working interest WI_{opt} . the decrease in the risk adjusted value RAV continues to negative. However, with increasing working interest above the optimum the risk adjusted values starts to decrease.

Also, it is important to note from the plot that for project with RT=$75MM at up to 100% participation level (working interest) the risk adjusted value RAV is still positive this implies that a full participation of such prospect will not be regarded as too risky by the decision maker since it is still within the company's risk capacity while at risk tolerance of $50MM above 80% working interest, the RAV tend towards negative and becomes too risky to undertake. In contrast, at risk tolerance of $20MM above 30% working interest the RAV is already negative making the project too risky for the firm at participation levels greater than 30%.

Analysis of Net Present Values NPV (V)

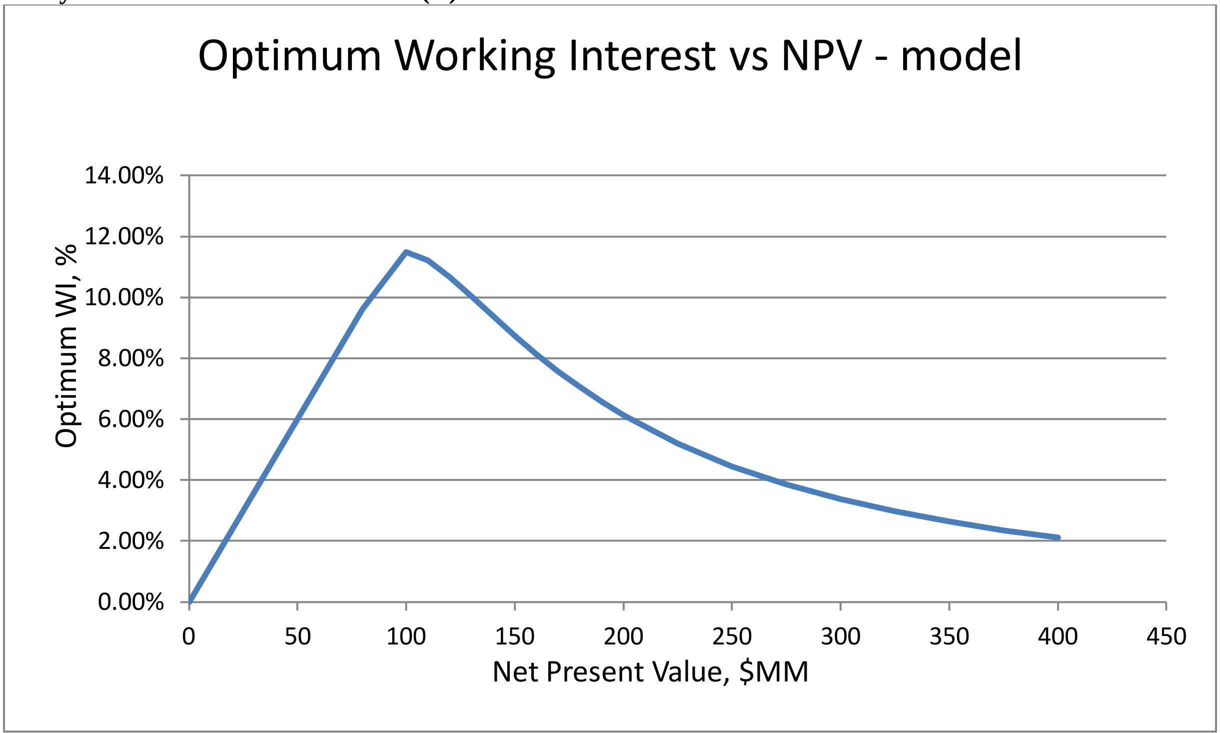

FIGURE 4 working interest, WI(%) versus *Net present value NPV* in $MM for model
***Net Present Value represented as 'V' on the table above.**

From the graph above it can seen that the optimum working interest increases with increase in the Net Present Value NPV and then reaches a certain maximum value and start to fall which is obviously an anomaly to the rule of thumb for investor behavior in an high gain situations. In high gain situations, the Optimum working Interest should increase with increasing gains i.e the more the gains, the more Investor should undertake. A possible explanation to this anomaly is that the Hyperbolic Models are based on minimizing variances, an index of uncertainty.

COMPARISON OF MODEL TO LERCHE MODEL
Comparison for risk adjusted values RAV

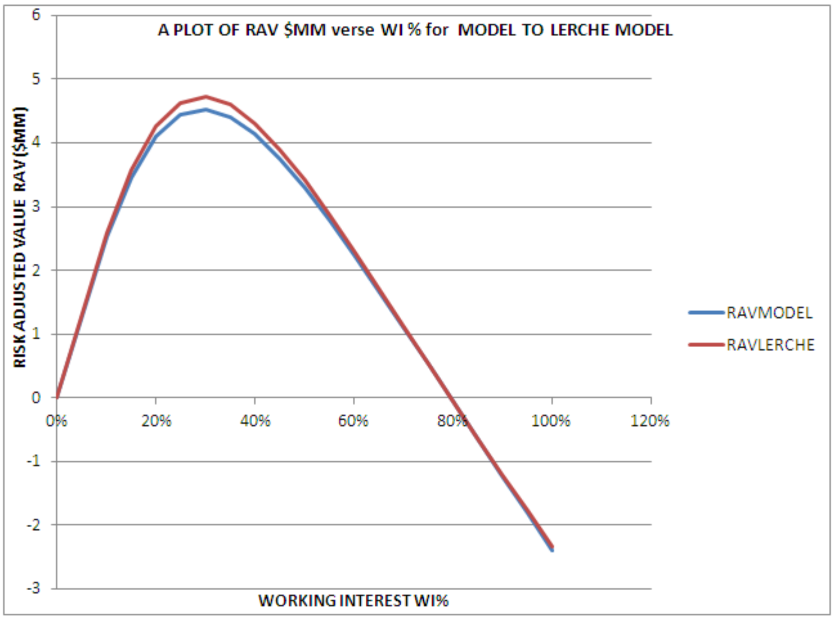

Figure 5 Comparison of Model and Lerche RAV vs WI using the parameters illustrated in figure 2b

At a fixed Risk Tolerance, comparison of the plot in figure 6 shows that model is more conservative than Lerche model in its estimates of risk adjusted values RAV as working interest increases i.e. Lerche model is more exaggerative. The maximum value of RAV which gives the optimum working interest tends to be achieved faster for model compare to that of Lerche model. Above the optimum working (RAV max), the RAV tend to decrease faster with working interest for this model comparable to Lerche model.

Justification of model's smaller values of WI_{opt}

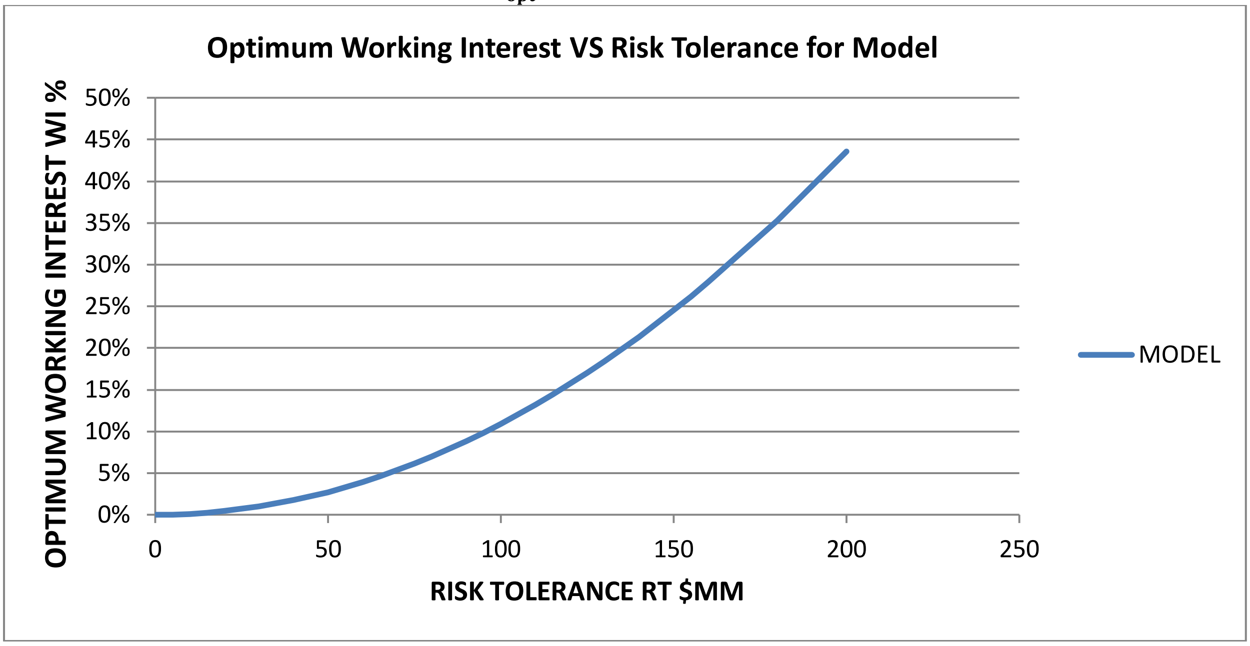

Figure 6a A plot of WI_{opt} vs RT for Model using the parameters illustrated in figure 2b

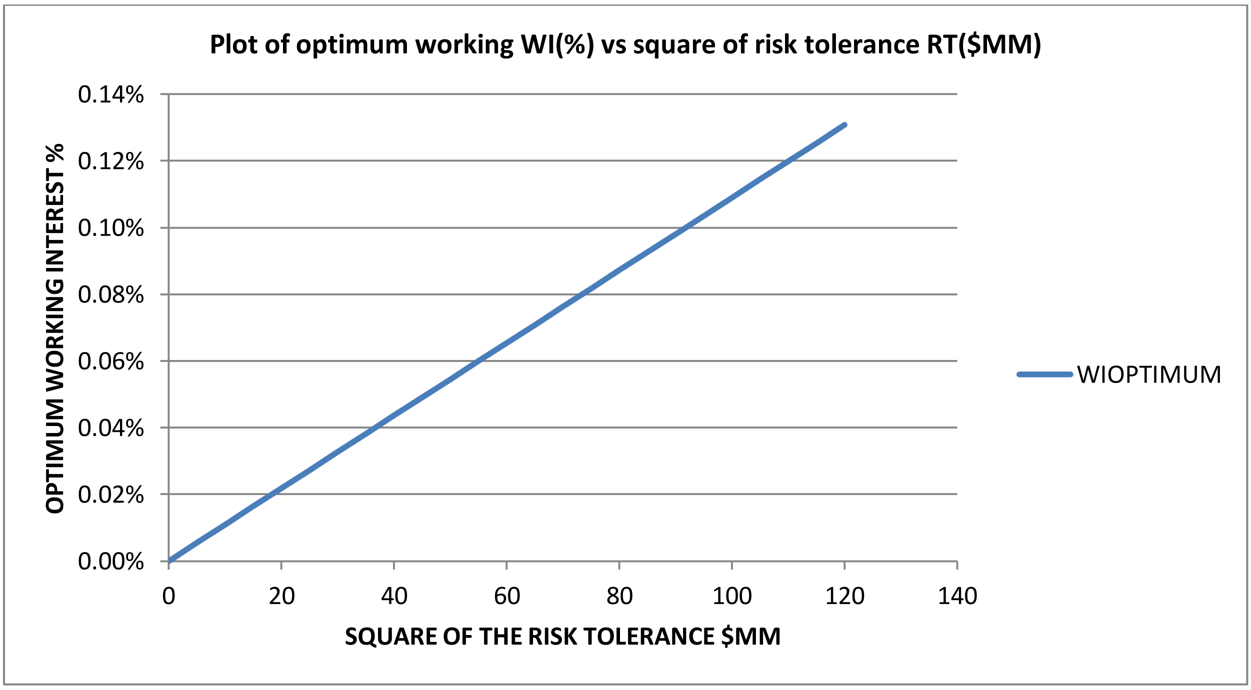

Figure 6b Plot of optimum working WI (%) vs square of risk tolerance RT ($MM)

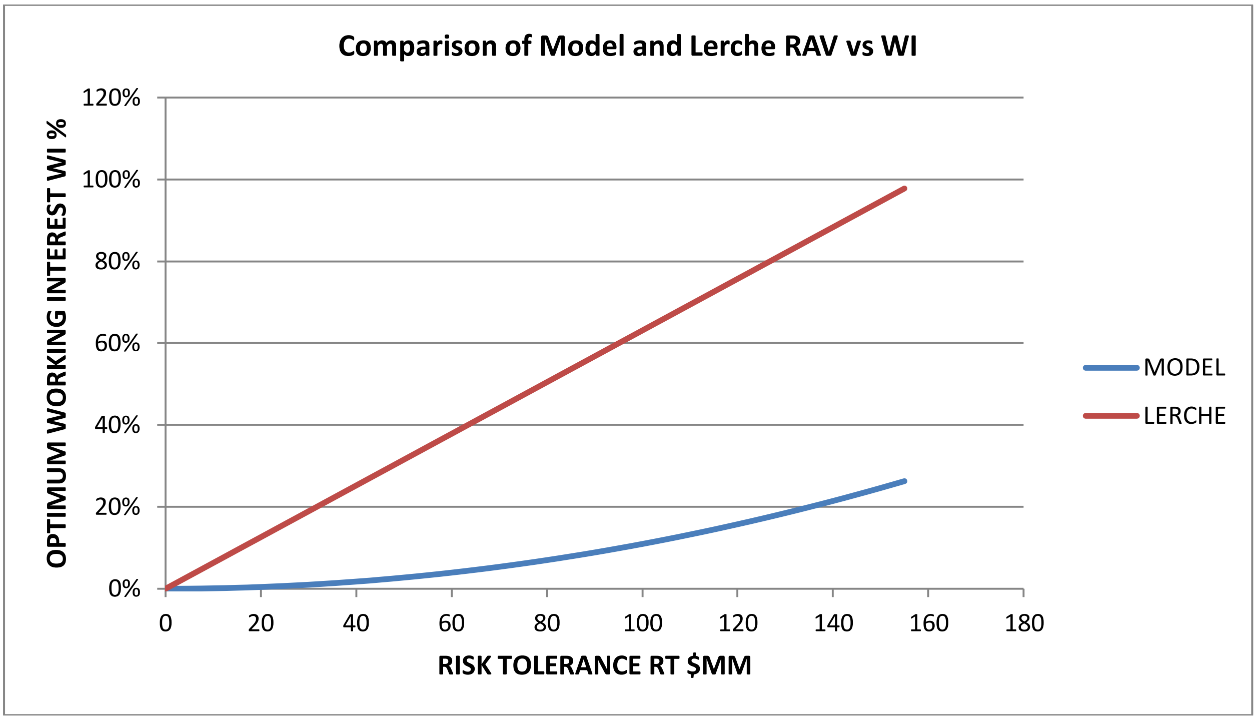

Figure 6c Comparison of Model and LercheWI_{opt} vs RT using the parameters illustrated in figure 2b

A careful study of the results given in the numerical illustrations in section of discussion and the plots shown in the figures above, it can be observed that the values obtained for optimum working interest WI_{opt} for a given risk tolerance RT is small when compare to Lerche model. This can be better explained mathematical this way;

For instance, consider the equation;

$Y = X^2K_1$ ……………………………………………………………………………………………… 2-11

$\therefore\ K_1 = \frac{Y}{X^2}$……………………………………………………………………………………………2-12

$Y = XK_2$ …………………………………………………… ………………………………………2-13

$K_2 = \frac{Y}{X}$ ……………………………………………………………………………………………………..2-14

where Y is between 0 and 1, X is a positive variable while K_1&K_2 are constant of proportionality.

From the equation 2-12 and 2-14 K_2 is greater than K_1

It is this constant of proportionality K_1 and K that multiplies with the variables X^2 and X to obtain Y, hence for equation 2-11 Y will be small and large for equation 2-13 i.e small value of K_1 yield small values of Y in comparison to larger value K_2 which invariably yield larger Y.

Now when the equation 2-11 is compared to the optimum working interest equation

$Y = X^2K_1$ and $WI_{opt} = \frac{RT^2\left((P_sV)^{\frac{1}{2}}-(P_fC)^{\frac{1}{2}}\right)}{\left((P_sV)^{\frac{1}{2}}V^2-(P_fC)^{\frac{1}{2}}C^2\right)}$

Y=WI_{opt} X=RT and $K_1 = \frac{\left((P_sV)^{\frac{1}{2}}-(P_fC)^{\frac{1}{2}}\right)}{\left((P_sV)^{\frac{1}{2}}V^2-(P_fC)^{\frac{1}{2}}C^2\right)}$

$\therefore\ WI_{opt} = RT^2K_1$…………………………………………………………………..…………....2-15

Since from equation 2-11 K_1 small, therefore the values of WI_{opt} will be small

In contrast, for Lerche model where;

$Y = XK_2$and $WI_{opt} = \frac{RT}{2C}\ln\left(\frac{4P_sV}{P_fC}\right)$

$Y=WI_{opt}$, X=RT and $K_2 = \frac{1}{2C}\ln\left(\frac{4P_sV}{P_fC}\right)$

$\therefore\ WI_{opt} = RTK_2$…………………………………………………………………………2-16

Since K_2 is larger than K_1, the values of WI_{opt} for equation 4.16 will always be larger than that obtained in equation 2-15.

NOTE:

- From the explanation above, the constant of proportionality K gives the reason why the optimum working interest WI_{opt} for model is and will always be smaller than that obtained from Lerche model, hence, justifies the smaller values of optimum working interest WI_{opt} obtained for model in comparison to Lerche model shown in both numerical and graphical illustrations. Same explanation applies to the breakeven working interest $\mathrm{WI}_{breakeven}$

Comparison for Net Present Values NPV

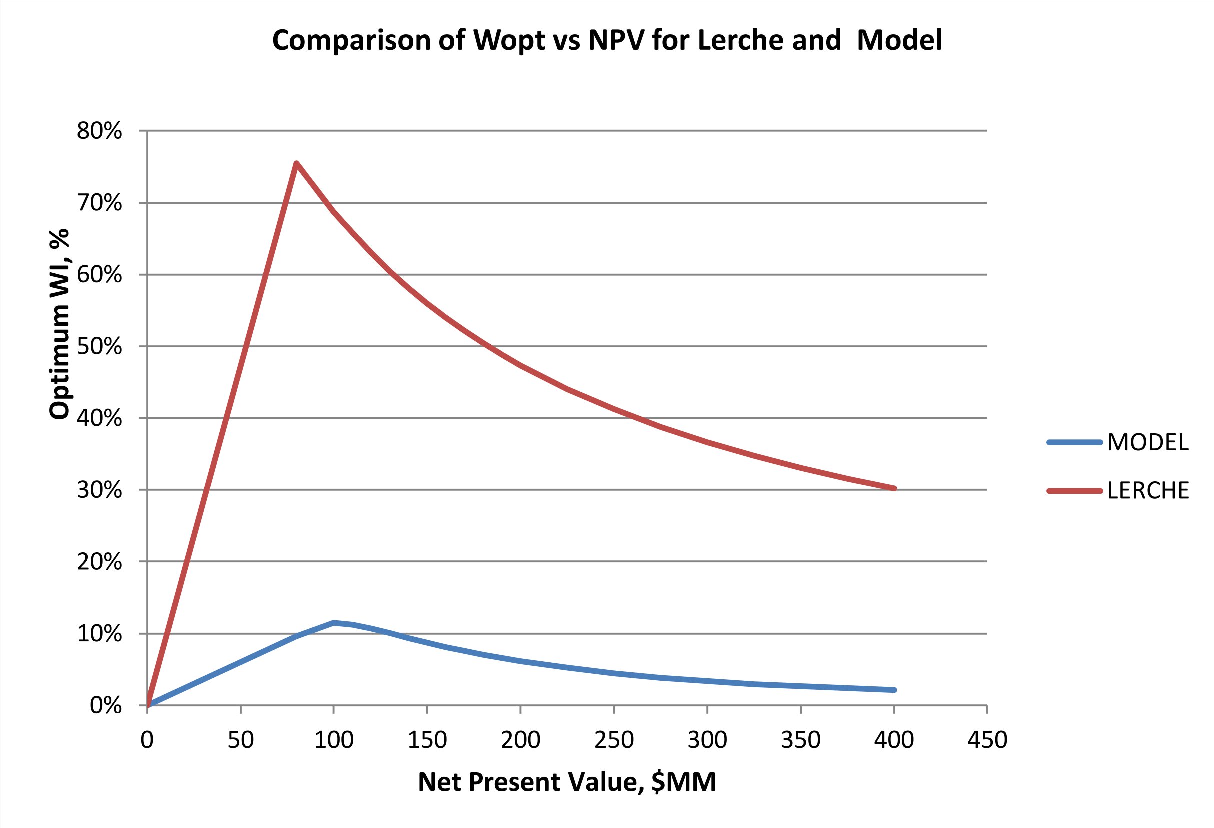

Figure 7 Comparison of Model and LercheWI_{opt} vs NPV using the parameters illustrated in figure

IV. CONCLUSIONS

The following conclusions can be drawn from this study.

This work investigated the hyperbolic tangent utility function model in accounting for individual investor risk preferences and explored the use of hyperbolic inversion of the hyperbolic utility function obtained in estimating risk adjusted value RAV.

It determines the optimum and breakeven working interest. It also shows how the optimum working interest changes with cost of investment and net present value NPV factor of a project.

Overall, it was discovered that optimum working interest is only very sensitive when the inputs are overly optimistic this occurs when probability of success (Ps) is too high, probability of failure (Pf) is too low.

V. RECOMMENDATIONS

- Further work should be done how risk adjusted values RAV may be used by government to optimize the size of individual blocks in licensing offerings.
- On a National Level, it will be interesting to conduct a Risk Adjusted Value Analysis of the current Joint Venture participation arrangements (60/40) of the Oil Nationals like the NLNG and determine the impact on Government/Operator profitability.
- The use of the hyperbolic model in accounting for individual investor risk preferences has not been rigorously investigated as Cozzolino did for the Exponential Model in his 1977 study.
- Further work is required to determine whether the risk adjusted value RAV for Hyperbolic model is a better predictor for market values than Exponential model.

- There is also the need for more model other than Exponential and Hyperbolic model to provide alternatives tools for decision makers to select from.

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