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Applications of modified Adomian decomposition methods to systems of nonlinear partial differential equations

AbdallahHabila Ali[1], AbrarYousif Elriyah[2]
Sudan University of Science and Technology, College of Science, Department of Mathematics (Sudan)[1]
Mashreq University, College of Science, Department of Mathematics (Sudan) [2]

Abstract: *In this paper, we apply the Adomian decomposition method (ADM) and Modified decomposition method(MDM) to four different types of nonlinear partial differential equations(PDEs). The proposed Adomian and Modified decomposition methods was applied to reformulated first and second order initial value problems, which leads the solution in terms of transformed variables, and the series solution will be obtained by making use of the inverse operator. The results indicate these methods to be very effective and simple.*

Keywords: *Systems of nonlinear partial differential equations, Adomian decomposition method , Modified decomposition method.*

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I. Introduction

The Adomian decomposition method has been receiving much attention in recent years in applied mathematics in general, and in the area of series solutions in particular[1-5]. The method proved to be powerful, effective, and can easily handle a wide class of linear or nonlinear, ordinary or partial differential equations, and linear and nonlinear integral equations [6-10]. The Adomian decomposition method was introduced and developed by George Adomian in [10-18] and is well addressed in the literature.

This paper is arranged as follows. In Section 2, the, Adomian decomposition method. In Section 3, The Modified Decomposition Method. In Section 4, numerical examples. The conclusions appear in Section 5.

II. The Adomian decomposition method

In this section of nonlinear partial differential equations will be examined by using Adomian decomposition method. Systems of nonlinear partial differential equations arise in many scientific models such as the propagation of shallow water waves and the Brusselator model of chemical reaction-diffusion model. To achieve our goal in handling systems of nonlinear partial differential equations , we write a system in an operator form by

$$L_t u + L_x v + N_1(u,v) = g_1$$

$$L_t u + L_x v + N_2(u,v) = g_2 \qquad (1)$$

With initial data

$$u(x,0) = f_1(x),$$

$$v(x,0) = f_2(x), \qquad (2)$$

Where $L_t$and L_x are considered, without loss of generality, first order partial differential operators,N_1 and N_2 are nonlinear operators, and $g_1$and g_2 are source terms. Operating with the integral operator L_t^{-1} to the system (1) and using the initial data (2) yields

$$u(x,t) = f_1(x) + L_t^{-1} g_1 - L_t^{-1} L_x v - L_t^{-1} N_1(u,v),$$

$$v(x,t) = f_2(x) + L_t^{-1} g_2 - L_t^{-1} L_x v - L_t^{-1} N_2(u,v), \qquad (3)$$

The linear unknown functions $u(x,t)$ and $v(x,t)$ can be decomposed by infinite series of components

$$u(x,t)=\sum_{n=0}^{\infty}u_n\,(x,t),$$

$v(x,t)=\sum_{n=0}^{\infty}v_n\,(x,t)$(4)

However, the nonlinear operators $N_1(u,v)$ and $N_2(u,v)$ should be represented by using the infinite series of the so-called Adomian polynomials A_n and B_n
As follows:

$$N_1(u,v)=\sum_{n=0}^{\infty}A_n,$$

$N_2(u,v)=\sum_{n=0}^{\infty}B_n,$ (5)

Where $u_n(x,t)$ and $v_n(x,t), n\geq 0$ are the components of $u(x,t)$ and $v(x,t)$ respectively that will be recurrently determined, and A_n and $B_n, n\geq 0$ are Adomian polynomials that can be generated for all forms of nonlinearity. Substituting (4) and (5) into (3) gives

$$\sum_{n=0}^{\infty}u_n\,(x,t)=f_1(x)+L_t^{-1}g_1-L_t^{-1}L_x(\sum_{n=0}^{\infty}v_n)-L_t^{-1}(\sum_{n=0}^{\infty}A_n),$$

$\sum_{n=0}^{\infty}v_n\,(x,t)=f_2(x)+L_t^{-1}g_2-L_t^{-1}L_x(\sum_{n=0}^{\infty}u_n)-L_t^{-1}(\sum_{n=0}^{\infty}B_n).$ (6)

Two recursive relations can be constructed from (1.6) given by

$$u_0(x,t)=f_1(x)+L_t^{-1}g_1,$$

$u_{k+1}(x,t)=-L_t^{-1}(L_xv_k)-L_t^{-1}(A_k),$ $k\geq 0,$ (7)

$$v_0(x,t)=f_2(x)+L_t^{-1}g_2,$$

$v_{k+1}(x,t)=-L_t^{-1}(L_xu_k)-L_t^{-1}(B_k),$ $k\geq 0,$ (8)

It is an essential feature of the decomposition method that the zeroth components $u_0(x,t)$and$v_0(x,t)$ are defined always by all terms that arise from initial data and from integrating the source terms. Having defined the zeroth pair (u_0,v_0), the remaining pair $(u_k,v_k)k\geq 1$ can be obtained in a recurrent manner by using (7) and (8) . Additional pairs for the decomposition series solutions normally account for higher accuracy. Having determined the components of $u(x,t)$and (x,t) , the solution (u,v) of the system follows immediately in the form of a power series expansion upon using (4).

III. The Modified Decomposition Method

The modified decomposition method will further accelerate the convergence of the series solution. It is to be noted that the modified decomposition method will be applied, wherever it is appropriate, to all partial differential equations of many order. To give a clear description of the technique, we consider the partial differential equation in an operator form

$$Lu+Ru=g, (9)$$

Where L is the highest order derivative, R is a linear differential operator of less order or equal order to L, and g is the source term. Operating with the inverse operator on (9) we obtain

$$u=f-L^{-1}(Ru), \quad (10)$$

Where f represents the terms arising from the given initial condition and form grating the source term g. Define the solution u as an infinite sum of components defined by

$u=\sum_{n=0}^{\infty}u_n$(11)

The aim of the decomposition method is to determine the components $u_n, n\geq 0$
Recurrently and elegantly. To achieve this goal, the decomposition method admits the use of the recursive relation

$$u_0=f,$$

$u_{k+1}=-L^{-1}(Ru_k),\quad k\geq 0.$ (12)

In view of (12), the components $u_n, n\geq 0$ are readily obtained.

The modified decomposition method introduces a slight variation to the recursive relation(12) that will lead to the determination of the components of u in a faster and easier way. For specific cases, the function f can be set as the sum of two partial functions, namely $f_1$and f_2.In other words, we can set

$$f=f_1+f_2 \quad (13)$$

Using (13), we introduce a qualitative change in the formation of the recursive relation (12). To reduce the size of calculations, we identify the zeroth component u_0 by one part of f , namely $f_1$or f_2. The other part of f can be added to the componentu_1 among other terms. I n other words, the modified recursive relation can be identified by

$$u_0 = f_1,$$
$$u_1 = f_2 - L^{-1}(Ru_0),$$
$$u_{k+1} = -L^{-1}(Ru_k), k \geq 1. \qquad (14)$$

Tow important remarks related to the modified method can be made here.

First, by proper selection of the functions $f_1$and f_2, the exact solution u may be obtained by using very few iterations, and sometimes by evaluating only two components. The success of this modification depends only on the choice of $f_1$and f_2, and this can be made through trials . Second , if f consists of one term only, the standard decomposition method should be employed in this case.

IV. Numerical Examples

Example 1. Consider the following nonlinear system:

$$u_t + v_x w_y - v_y w_x = -u,$$
$$v_t + w_x u_y + w_y u_x = v,$$
$$w_t + u_x v_y + u_y v_x = w, \quad (15)$$

With the initial conditions

$$u(x,y,0) = e^{x+y}, \quad v(x,y,0) = e^{x-y}, w(x,y,0) = e^{-x+y} \qquad (16)$$

Solution.

Operating with L_t^{-1},we obtain

$$u(x,y,t) = e^{x+y} + L_t^{-1}\left(v_y w_x - v_x w_y - u\right),$$
$$v(x,y,t) = e^{x-y} + L_t^{-1}\left(v - w_x u_y - w_y u_x\right),$$
$$w(x,y,t) = e^{-x+y} + L_t^{-1}\left(w - u_x v_y - u_y v_x\right) \qquad (17)$$

Substituting the decomposition representations for linear and nonlinear into (17) yields

$$\sum_{n=0}^{\infty} u_n(x,t) = e^{x+y} + L_t^{-1}\left(\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n - \sum_{n=0}^{\infty} u_n\right),$$

$$\sum_{n=0}^{\infty} v_n(x,t) = e^{x-y} + L_t^{-1}\left(\sum_{n=0}^{\infty} v_n - \sum_{n=0}^{\infty} C_n - \sum_{n=0}^{\infty} D_n\right),$$

$$\sum_{n=0}^{\infty} w_n(x,t) = e^{-x+y} + L_t^{-1}\left(\sum_{n=0}^{\infty} w_n - \sum_{n=0}^{\infty} E_n - \sum_{n=0}^{\infty} F_n\right) (18)$$

Where A_n, B_n, C_n, D_n, E_n and $F_n$are Adomian polynomials for the nonlinear terms $v_y w_x, v_x w_y, w_x u_y, w_y u_x, u_x v_y$ and $u_y v_x$ respectively.

Three recursive relation can be constructed from equation (18) given by:

$$u_0 = e^{x+y},$$
$$u_{k+1} = L_t^{-1}(A_k - B_K - u_K), \quad k \geq 0 \qquad (19)$$
$$v_0 = e^{x-y},$$
$$v_{k+1} = L_t^{-1}(v_k - C_K - D_K), \quad k \geq 0 \qquad (20)$$

and

$$w_0 = e^{-x+y}$$
$$w_{k+1} = L_t^{-1}(w_k - E_K - F_K), \quad k \geq 0 \qquad (21)$$

We list the first three Adomian polynomials as follows:

For $v_y w_x$ we find

$$A_0 = v_{0_y} w_{0_x},$$
$$A_1 = v_{1_y} w_{0_x} + v_{0_y} w_{1_x},$$
$$A_2 = v_{2_y} w_{0_x} + v_{1_y} w_{1_x} + v_{0_y} w_{2_x},$$

For $v_x w_y$ we find

$$B_0 = v_{0_x} w_{0_y},$$
$$B_1 = v_{1_x} w_{0_y} + v_{0_x} w_{1_y},$$
$$B_2 = v_{2_x} w_{0_y} + v_{1_x} w_{1_y} + v_{0_x} w_{2_y},$$

For $w_x u_y$ we find

$$C_0 = w_{0_x} u_{0_y},$$
$$C_1 = w_{1_x} u_{0_y} + w_{0_x} u_{1_y},$$

$$C_2 = w_{2_x}u_{0_y} + w_{1_x}u_{1_y} + w_{0_x}u_{2_y},$$

For $w_y u_x$ we find

$$D_0 = w_{0_y}u_{0_x},$$
$$D_1 = w_{1_y}u_{0_x} + w_{0_y}u_{1_x},$$
$$D_2 = w_{2_y}u_{0_x} + w_{1_y}u_{1_x} + w_{0_y}u_{2_x},$$

For $u_x v_y$ we find

$$E_0 = u_{0_x}v_{0_y},$$
$$E_1 = u_{1_x}v_{0_y} + u_{0_x}v_{1_y},$$
$$E_2 = u_{2_x}v_{0_y} + u_{1_x}v_{1_y} + u_{0_x}v_{2_y},$$

For $u_y v_x$ we find

$$F_0 = u_{0_y}v_{0_x},$$
$$F_1 = u_{1_y}v_{0_x} + u_{0_y}v_{1_x},$$
$$F_2 = u_{2_y}v_{0_x} + u_{1_y}v_{1_x} + u_{0_y}v_{2_x},$$

Using the derived Adomian polynomials into equations (19), (20) and (21), we obtain:

$$u_0 = e^{x+y}, \qquad v_0 = e^{x-y}, \qquad w_0 = e^{-x+y}$$
$$u_1 = L_t^{-1}(A_0 - B_0 - u_0) = -te^{x+y}$$
$$v_1 = L_t^{-1}(v_0 - C_0 - D_0) = te^{x-y}$$
$$w_1 = L_t^{-1}(w_0 - E_0 - F_0) = te^{-x+y}$$
$$u_2 = L_t^{-1}(A_1 - B_1 - u_1) = \frac{t^2}{2!}e^{x+y}$$
$$v_2 = L_t^{-1}(v_1 - C_1 - D_1) = \frac{t^2}{2!}e^{-x+y}$$
$$w_2 = L_t^{-1}(w_1 - E_1 - F_1) = \frac{t^2}{2!}e^{-x+y}$$
$$u_3 = L_t^{-1}(A_2 - B_2 - u_2) = -\frac{t^3}{3!}e^{x+y}$$
$$v_3 = \frac{t^3}{3!}e^{x-y}, w_3 = \frac{t^3}{3!}e^{-x+y}$$

And so on,

The solutions $u(x,y,t), v(x,y,t)$ and $w(x,y,t)$ in a series form are given by:

$u(x,y,t) = e^{x+y} - te^{x+y} + \frac{t^2}{2!}e^{x+y} - \frac{t^3}{3!}e^{x+y} + \cdots = e^{x+y-t}$(22)

$v(x,y,t) = e^{x-y} + te^{x-y} + \frac{t^2}{2!}e^{x-y} + \frac{t^3}{3!}e^{x-y} + \cdots = e^{x-y+t}$(23)

$w(x,y,t) = e^{-x+y} + te^{-x+y} + \frac{t^2}{2!}e^{-x+y} + \frac{t^3}{3!}e^{-x+y} + \cdots = e^{-x+y+t}$(24)

Which are the exact solutions.

Example 2. Consider the following nonlinear system:

$u_t + u_x v_x - u_y v_y + u = 0,$

$$v_t + v_x w_x - v_y w_y - v = 0,$$

$w_t + w_x u_x + w_y u_y - w = 0,$(25)

With the initial conditions

$u(x,y,0) = e^{x+y}, \quad v(x,y,0) = e^{x-y}, w(x,y,0) = e^{-x+y}$(26)

Solution.

Operating with L_t^{-1},we obtain

$$u(x,y,t) = e^{x+y} + L_t^{-1}(u_y v_y - u_x v_x - u),$$
$$v(x,y,t) = e^{x-y} + L_t^{-1}(v + v_y w_y - v_x w_x),$$

$w(x,y,t) = e^{-x+y} + L_t^{-1}(w - w_x u_x - w_y u_y),$ (27)

Substituting the decomposition representations for linear and nonlinear into (27) yields

$$\sum_{n=0}^{\infty} u_n(x,t) = e^{x+y} + L_t^{-1}\left(\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n - \sum_{n=0}^{\infty} u_n\right),$$

$$\sum_{n=0}^{\infty} v_n\,(,t) = e^{x-y} + L_t^{-1}\left(\sum_{n=0}^{\infty} v_n + \sum_{n=0}^{\infty} C_n - \sum_{n=0}^{\infty} D_n\right),$$

$\sum_{n=0}^{\infty} w_n\,(,t) = e^{-x+y} + L_t^{-1}(\sum_{n=0}^{\infty} w_n - \sum_{n=0}^{\infty} E_n - \sum_{n=0}^{\infty} F_n)$(28)
Where A_n, B_n, C_n, D_n, E_n and F_n are Adomian polynomials for the nonlinear terms $u_y v_y, u_x v_x, v_y w_y, v_x w_x, u_x w_x$ and $w_y u_y$ respectively.
Three recursive relation can be constructed from equation (28) given by:

$$u_0 = e^{x+y},$$

$$u_{k+1} = L_t^{-1}(A_k - B_K - u_K),\ \ k \geq 0 \tag{29}$$

$$v_0 = e^{x-y},$$

$$v_{k+1} = L_t^{-1}(v_k + C_K - D_K),\ \ k \geq 0 \tag{30}$$

and

$$w_0 = e^{-x+y}$$

$$w_{k+1} = L_t^{-1}(w_k - E_K - F_K),\ \ k \geq 0 \tag{31}$$

We list the first three Adomian polynomials as follows:
For $u_y v_y$ we find

$$A_0 = u_{0_y} v_{0_y},$$
$$A_1 = u_{1_y} v_{0_y} + u_{0_y} v_{1_y},$$
$$A_2 = u_{2_y} v_{0_y} + u_{1_y} v_{1_y} + u_{0_y} v_{2_y},$$

For $u_x v_x$ we find

$$B_0 = u_{0_x} v_{0_x},$$
$$B_1 = u_{1_x} v_{0_x} + u_{0_x} v_{1_x},$$
$$B_2 = u_{2_x} v_{0_x} + u_{1_x} v_{1_x} + u_{0_x} v_{2_x},$$

For $v_y w_y$ we find

$$C_0 = v_{0_y} w_{0_y},$$
$$C_1 = v_{1_y} w_{0_y} + v_{0_y} w_{1_y},$$
$$C_2 = v_{2_y} w_{0_y} + v_{1_y} w_{1_y} + v_{0_y} w_{2_y}$$

For $v_x w_x$ we find

$$D_0 = v_{0_x} w_{0_x},$$
$$D_1 = v_{1_x} w_{0_x} + v_{0_x} w_{1_x},$$
$$D_2 = v_{2_x} w_{0_x} + v_{1_x} w_{1_x} + v_{0_x} w_{2_x}$$

For $u_x w_x$ we find

$$E_0 = u_{0_x} w_{0_x},$$
$$E_1 = u_{1_x} w_{0_x} + u_{0_x} w_{1_x},$$
$$E_2 = u_{2_x} w_{0_x} + u_{1_x} w_{1_x} + u_{0_x} w_{2_x}$$

For $w_y u_y$ we find

$$F_0 = w_{0_y} u_{0_y},$$
$$F_1 = w_{1_y} u_{0_y} + w_{0_y} u_{1_y},$$
$$F_2 = w_{2_y} u_{0_y} + w_{1_y} u_{1_y} + w_{0_y} u_{2_y}$$

Using the derived Adomian polynomials into equations (29), (30) and (31), we obtain:

$$u_0 = e^{x+y}, \qquad v_0 = e^{x-y}, \qquad w_0 = e^{-x+y}$$
$$u_1 = L_t^{-1}(A_0 - B_0 - u_0) = -te^{x+y}$$
$$v_1 = L_t^{-1}(v_0 + C_0 - D_0) = te^{x-y}$$
$$w_1 = L_t^{-1}(w_0 - E_0 - F_0) = te^{-x+y}$$
$$u_2 = L_t^{-1}(A_1 - B_1 - u_1) = \frac{t^2}{2!}e^{x+y}$$

$$v_2 = L_t^{-1}(v_1 + C_1 - D_1) = \frac{t^2}{2!}e^{x-y}$$
$$w_2 = L_t^{-1}(w_1 - E_1 - F_1) = \frac{t^2}{2!}e^{-x+y}$$
$$u_3 = L_t^{-1}\left(-\frac{t^2}{2!}e^{x+y}\right) = -\frac{t^3}{3!}e^{x+y}$$
$$v_3 = L_t^{-1}\left(\frac{t^2}{2!}e^{x-y}\right) = \frac{t^3}{3!}e^{x-y}, w_3 = L_t^{-1}\left(\frac{t^2}{2!}e^{-x+y}\right) = \frac{t^3}{3!}e^{-x+y}$$

And so on,

The solutions $u(x,y,t), v(x,y,t)$ and $w(x,y,t)$ in a series form are given by:

$u(x,y,t) = e^{x+y} - te^{x+y} + \frac{t^2}{2!}e^{x+y} - \frac{t^3}{3!}e^{x+y} + \cdots = e^{x+y-t}$(32)

$v(x,y,t) = e^{x-y} + te^{x-y} + \frac{t^2}{2!}e^{x-y} + \frac{t^3}{3!}e^{x-y} + \cdots = e^{x-y+t}$(33)

$w(x,y,t) = e^{-x+y} + te^{-x+y} + \frac{t^2}{2!}e^{-x+y} + \frac{t^3}{3!}e^{-x+y} + \cdots = e^{-x+y+t}$ (34)

Which are the exact solutions.

Example 3. Consider the following nonlinear system:

$u_t + u_x v_x - w_y = 1,$

$$v_t + v_x w_x + u_y = 1,$$

$w_t + w_x u_x - v_y = 1,$(35)

With the initial conditions

$u(x,y,0) = x + y, \quad v(x,y,0) = x - y, w(x,y,0) = -x + y$ (36)

Solution.

Operating with L_t^{-1},we obtain

$$u(x,y,t) = x + y + t + L_t^{-1}(w_y - u_x v_x),$$
$$v(x,y,t) = x - y + t + L_t^{-1}(-u_y - v_x w_x),$$

$w(x,y,t) = -x + y + t + L_t^{-1}(v_y - w_x u_x),$ (37)

Substituting the decomposition representations for linear and nonlinear into (37) yields

$$\sum_{n=0}^{\infty} u_n(x,y,t) = x + y + t + L_t^{-1}\left(\sum_{n=0}^{\infty} w_{n_y} - \sum_{n=0}^{\infty} A_n\right),$$
$$\sum_{n=0}^{\infty} v_n(x,y,t) = x - y + t - L_t^{-1}\left(\sum_{n=0}^{\infty} u_{n_y} + \sum_{n=0}^{\infty} B_n\right),$$

$\sum_{n=0}^{\infty} w_n(x,y,t) = -x + y + t + L_t^{-1}\left(\sum_{n=0}^{\infty} v_{n_y} - \sum_{n=0}^{\infty} C_n\right)$(38)

Where A_n, B_n, and C_n are Adomian polynomials for the nonlinear terms $u_x v_x, v_x w_x$,and $w_x u_x$ respectively.

Three recursive relation can be constructed from equation (38) given by:

$$u_0 = x + y,$$
$$u_1 = t + L_t^{-1}\left(w_{0_y} - A_0\right)$$

$u_{k+1} = L_t^{-1}\left(w_{k_y} - A_k\right), \ k \geq 1$ (39)

$$v_0 = x - y,$$
$$v_1 = t - L_t^{-1}\left(u_{0_y} + B_0\right)$$

$v_{k+1} = -L_t^{-1}\left(u_{k_y} + B_k\right), \ k \geq 1$ (40)

and

$$w_0 = -x + y$$
$$w_1 = t + L_t^{-1}\left(v_{0_y} - C_0\right)$$

$w_{k+1} = L_t^{-1}\left(v_{k_y} - C_K\right), \ k \geq 1$ (41)

We list the Adomian polynomials as follows:

For $u_x v_x$ we find

$$A_0 = u_{0_x} v_{0_x},$$

$$A_1 = u_{1_x} v_{0_x} + u_{0_x} v_{1_x},$$

For $v_x w_x$ we find

$$B_0 = v_{0_x} w_{0_x},$$
$$B_1 = v_{1_x} w_{0_x} + v_{0_x} w_{1_x},$$

For $w_x u_x$ we find

$$C_0 = w_{0_x} u_{0_x},$$
$$C_1 = w_{1_x} u_{0_x} + w_{0_x} u_{1_x},$$

Using the derived Adomian polynomials into equations (39), (40) and (41), we obtain:

$$u_0 = x + y, \qquad v_0 = x - y, \qquad w_0 = -x + y$$
$$u_1 = t + L_t^{-1}(1 - 1) = t$$
$$v_1 = t - L_t^{-1}(1 - 1) = t$$
$$w_1 = t + L_t^{-1}(-1 + 1) = t$$
$$u_2 = L_t^{-1}(0) = 0,$$
$$v_2 = -L_t^{-1}(0) = 0$$
$$w_2 = L_t^{-1}(0) = 0,$$
$$A_n = 0\,, n \geq 1$$
$$u_k = 0\,, k \geq 2$$
$$v_k = 0\,, k \geq 2$$
$$w_k = 0\,, k \geq 2$$

Which is leading to the exact solution

$$u(x, y, t) = x + y + t + 0\,,$$
$$v(x, y, t) = x - y + t + 0\,,$$
$$w(x, y, t) = -x + y + t + 0\,. \qquad (42)$$

Example 4. Consider the following nonlinear system:

$$u_t + u_y v_x = 1 + e^t,$$
$$v_t + v_y w_x = 1 - e^{-t},$$
$$w_t + w_y u_y = 1 - e^{-t}, \qquad (43)$$

With the initial conditions

$$u(x, y, 0) = 1 + x + y, \quad v(x, y, 0) = 1 + x - y, w(x, y, 0) = 1 - x + y \quad (44)$$

Solution.

Operating with L_t^{-1},we obtain

$$u(x, y, t) = 1 + x + y + t + e^t - L_t^{-1}(u_y v_x),$$
$$v(x, y, t) = 1 + x - y + t + e^{-t} - L_t^{-1}(v_y w_x),$$
$$w(x, y, t) = 1 - x + y + t + e^{-t} + L_t^{-1}(w_y u_y), \qquad (45)$$

Substituting the decomposition representations for linear and nonlinear into (46) yields

$$\sum_{n=0}^{\infty} u_n\,(x, y, t) = 1 + x + y + t + e^t - L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right),$$

$$\sum_{n=0}^{\infty} v_n\,(x, y, t) = 1 + x - y + t + e^{-t} - L_t^{-1}\left(\sum_{n=0}^{\infty} B_n\right),$$

$$\sum_{n=0}^{\infty} w_n\,(x, y, t) = 1 - x + y + t + e^{-t} - L_t^{-1}\left(\sum_{n=0}^{\infty} C_n\right) \quad (46)$$

Where $A_n, B_n,$ and C_n are Adomian polynomials for the nonlinear terms $u_y v_x, v_y w_x$,and $w_y u_y$ respectively. Three recursive relation can be constructed from equation (46) given by:

$$u_0 = 1 + x + y,$$
$$u_1 = t + e^t - L_t^{-1}(A_0)$$
$$u_{k+1} = -L_t^{-1}(A_k), \quad k \geq 1 \qquad (47)$$
$$v_0 = 1 + x - y,$$
$$v_1 = t + e^{-t} - L_t^{-1}(B_0)$$

$v_{k+1} = -L_t^{-1}(B_k), \ \ k \geq 1$ (48)
and

$$w_0 = -x + y + t$$
$$v_1 = t + e^{-t} - L_t^{-1}(C_0)$$

$w_{k+1} = -L_t^{-1}(C_K), \ \ k \geq 1$ (49)
We list the Adomian polynomials as follows:
For $u_y v_x$ we find

$$A_0 = u_{0_y} v_{0_x},$$
$$A_1 = u_{1_y} v_{0_x} + u_{0_y} v_{1_x},$$

For $v_y w_x$ we find

$$B_0 = v_{0_y} w_{0_x},$$
$$B_1 = v_{1_y} w_{0_x} + v_{0_y} w_{1_x},$$

For $w_y u_y$ we find

$$C_0 = w_{0_y} u_{0_y},$$
$$C_1 = w_{1_y} u_{0_y} + w_{0_y} u_{1_y},$$

Using the derived Adomian polynomials into equations (47), (48) and (49), we obtain:

$$u_0 = 1 + x + y, \quad v_0 = 1 + x - y, \quad w_0 = -x + y + t$$
$$u_1 = t + e^t - L_t^{-1}(1) = t + e^t - t = e^t$$
$$v_1 = t + e^{-t} - L_t^{-1}(1) = t + e^{-t} - t = e^{-t}$$
$$w_1 = t + e^{-t} - L_t^{-1}(1) = t + e^{-t} - t = e^{-t}$$
$$u_1 = -L_t^{-1}(A_1) = 0,$$
$$u_1 = -L_t^{-1}(B_1) = 0,$$
$$u_1 = -L_t^{-1}(C_1) = 0,$$

Accordingly, the solution of the system in a series form is given by
$u = 1 + x + y + e^t$
$v = 1 + x - y + e^{-t}$
$w = 1 - x + y + e^{-t}$ (50)

V. Conclusion

In this paper, we introduced of nonlinear partial differential equations, and solved them by using Adomian and Modified decomposition methods. These methods are very effective and accelerate the convergent of solution.

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