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Synthesis and Dynamic Simulation of an Offset Slider Crank Mechanism

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ABSTRACT:
Illustrates this paper synthesis and dynamic simulation of an offset slider crank mechanism. At the beginning, the length of the crank and connecting rod are mathematically proved based on the stroke length, advance to return ratio and offset height. The kinematic and dynamic analyses for links leading to spare matrices equations were done for position, velocity and acceleration the studies the dynamic forces at the joints of the mechanism are applied. The positions of all the links or elements in the mechanism for each increment of input motion are found, then the position equations versus time are differentiated.MATLAB gives the results in numerical form in this paper, the m-file codesgive the all the calculated values as output and it generates plots. A simulation model using MATLAB and SIMULINK are Produced for the synthesized mechanism. The unidentified parameters are solved by MATLAB and SIMULINK and displays those outcomes in graphical forms.

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I. THE OFFSET SLIDER-CRANK POSITION SOLUTION:

The following theoretical analysis was taken from Norton (1999). The vector loop approach is used and can be applied to a linkage containing sliders. The term offset means that the slider axis extended does not pass through the crack pivot. This linkage could be represented by only three position vectors, R_2, R_3 and R_s, but one of them (R_s) will be a vector of varying magnitude and angle. It will be easier to user four vectors R_1, R_2, R_3 and R_4 with R_1 arranged parallel to the axis of sliding and R_4 perpendicular. In effect the pair of vectors R_1 and R_4 are orthogonal components of the position vector R_s from the origin to the slider.

It simplifies the analysis to arrange one coordinate axis parallel to the axis of sliding. The variable-length, constant-direction vector R_1 then represents the slider position with magnitude d. The vector R_4 is orthogonal to R_1 and defines the constant magnitude offset of the linkage. The vectors R_2 and R_3 complete the vector loop. The coupler's position vector R_3 is placed with its root at the slider which then defines its angle α at position B. This arrangement of position vectors leads to a vector loop equation given by, see figure 1 for reference:

$$\vec{R}_2 + \vec{R}_3 = \vec{R}_4 + \vec{R}_1$$

Once these arbitrary choices are made it is crucial that the resulting algebraic signs be carefully observed in the equations or the results will be completely wrong. Letting the vector magnitude (link lengths) be represented by a, b, c and d as shown in figure 1, can substitute the complex number equivalents for the position vectors.

$$ae^{i\theta} + be^{i\alpha} = ce^{i\frac{3}{2}\pi} + de^{i0}$$

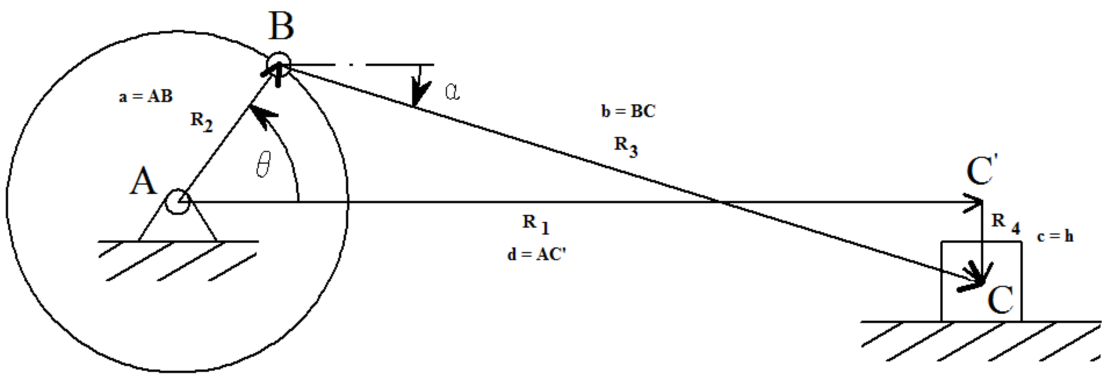


Figure 1: Position vector loop for offset slider-crank linkage.

Substitute the Euler equivalents:

$$ae^{i\theta} + be^{i\alpha} = ce^{i\frac{3}{2}\pi} + de^{i0}$$

$$a(cos\theta + i\sin\theta) + b(\cos\alpha + i\sin\alpha) = c\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right) + d(\cos 0 + i\sin 0)$$

Separate the real and imaginary components:
Real part (x component):

$$a\cos\theta + b\cos\alpha = c\cos\left(\frac{3}{2}\pi\right) + \ d\cos 0$$

Imaginary part (y component):

$$a\sin\theta + b\sin\alpha = c\sin\left(\frac{3}{2}\pi\right) + d\sin 0$$

Want to solve the two set equation simultaneously for the two unknowns, link length d and link angle α. The independent variable is crank angle θ. Link lengths a and b and the offset c are known.

$$a\cos\theta + b\cos\alpha = \ \ d$$
$$a\sin\theta + b\sin\alpha = -c$$

Solution for α and d are given by:

$$\alpha = \sin^{-1}\left(\frac{-c - a\sin\theta}{b}\right)$$

$$d = \ a\cos\theta + b\cos\alpha$$

In order to complete the design, taken into account the following specifications:

1. Stroke length of slider block $C \ = \ 100\ mm$
2. Advance-to-return time ratio, $T_R = 1.25$
3. Offset height $h \ = \ 30\ mm$

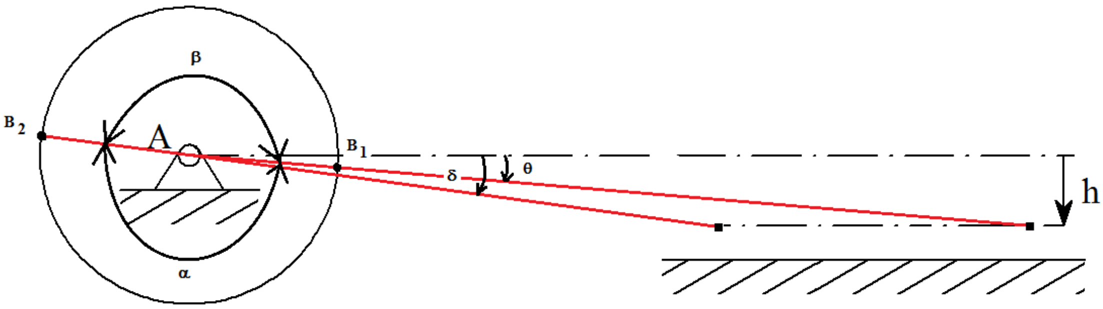


Figure 2. It shows the advance-to-return time ration,T_R.

Verified that the lengths of the crank ($a = AB = 47\,mm$) and the connecting rod ($b = BC = 105\,mm$) are satisfied. The following equations are derived from the figure 2.

$$\sin\delta = \frac{-h}{BC - AB}$$

$$\sin\theta = \frac{-h}{AB + BC}$$

$$\psi = \delta - \theta$$

The advance-to-return time ratio is defined as:

$$T_R = \frac{\alpha}{\beta} = \frac{\pi - \psi}{\pi + \psi} = 1.25$$

Thus,

$$\psi = -\frac{\pi}{9} \approx -20^0$$

On the other hand, from figure 2 and cosine law, got:

$$L^2 = (BC - AB)^2 + (AB + BC)^2 - 2(BC - AB)(AB + BC)\cos\psi$$

In addition, known that:

$$\psi = \delta - \theta$$

Thus,

$$\delta - \theta = \psi$$

$$\sin^{-1}\left(\frac{-h}{BC - AB}\right) - \sin^{-1}\left(\frac{-h}{AB + BC}\right) = \psi$$

With, $h = 30\,mm$, will have:

$$\sin^{-1}\left(\frac{-30}{BC - AB}\right) - \sin^{-1}\left(\frac{-30}{AB + BC}\right) = -\frac{\pi}{9}$$

Will take into account that:

$$L^2 = (BC - AB)^2 + (AB + BC)^2 - 2(BC - AB)(AB + BC)\cos\psi$$

With

$$\cos\psi = \cos\left(-\frac{\pi}{9}\right) \approx 0.93969$$

$$L^2 = (100\,mm)^2 = 10000\,mm^2$$

$$\chi_1 \equiv BC - AB$$

$$\chi_2 \equiv AB + BC$$

got:

$$\chi_1{}^2 + \chi_2{}^2 - 2\cos\left(-\frac{\pi}{9}\right)\chi_1\chi_2 - 10000 = 0$$

Solving for χ_1 as a function ofχ_2:

$\chi_1 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, $a = 1$, $b = -2\cos\left(-\frac{\pi}{9}\right)\chi_2$, $c = {\chi_2}^2 - 10000$

$$\chi_1 = \frac{2\cos\left(-\frac{\pi}{9}\right)\chi_2 \pm \sqrt{\left(2\cos\left(-\frac{\pi}{9}\right)\chi_2\right)^2 - 4({\chi_2}^2 - 10000)}}{2}$$

$$\chi_1 = \frac{2\cos\left(-\frac{\pi}{9}\right)\chi_2 \pm \sqrt{40000 - 4{\chi_2}^2\sin^2\left(-\frac{\pi}{9}\right)}}{2}$$

$$valid\ for\ 0 \le \chi_2 \le \frac{100}{\sin\left(\frac{\pi}{9}\right)}$$

In addition, will have the equation:

$$\sin^{-1}\left(\frac{-30}{\chi_1}\right) - \sin^{-1}\left(\frac{-30}{\chi_2}\right) = -\frac{\pi}{9}$$

Solved the unknown variables $\chi_1$and χ_2 graphically. Will take the positive solution for the quadratic equation then found that there is not solution for χ_2 and χ_1 as can be seen in figure 3.

$$f(\chi_2) = \sin^{-1}\left(\frac{-30}{\frac{2\cos\left(-\frac{\pi}{9}\right)\chi_2 + \sqrt{40000 - 4{\chi_2}^2\sin^2\left(-\frac{\pi}{9}\right)}}{2}}\right) - \sin^{-1}\left(\frac{-30}{\chi_2}\right) + \frac{\pi}{9} = 0$$

For this relation there is not solution for χ_2 andχ_1.

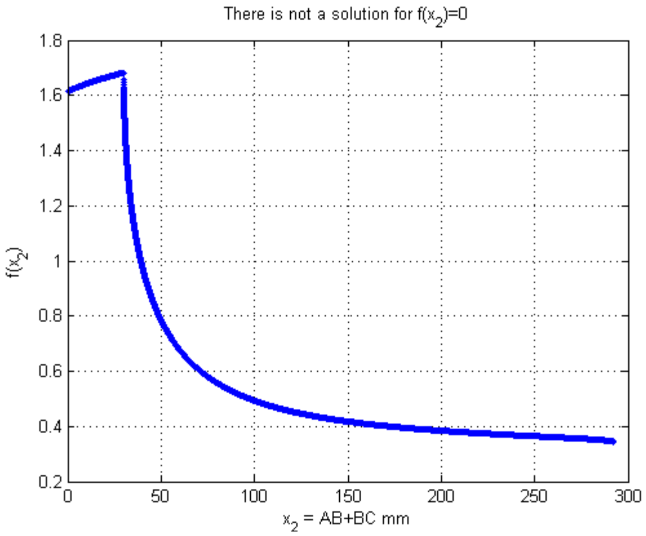


Figure 3: It shows the graphical method to find the solution forχ_2, however, didn't find any solution. have taken into account the positive root square in the quadratic equation.

On the other hand,take the negative solution for the quadratic equation and found that there is a solution for χ_2 and χ_1 as can be seen in figure 4 and figure 5.

$$g(\chi_2)^{-1} = \sin^{-1}\left(\frac{-30}{\frac{2\cos\left(-\frac{\pi}{9}\right)\chi_2 - \sqrt{40000 - 4{\chi_2}^2\sin^2\left(-\frac{\pi}{9}\right)}}{2}}\right) - \sin^{-1}\left(\frac{-30}{\chi_2}\right) + \frac{\pi}{9} = 0$$

For this relation there are solutions for both χ_2 andχ_1.

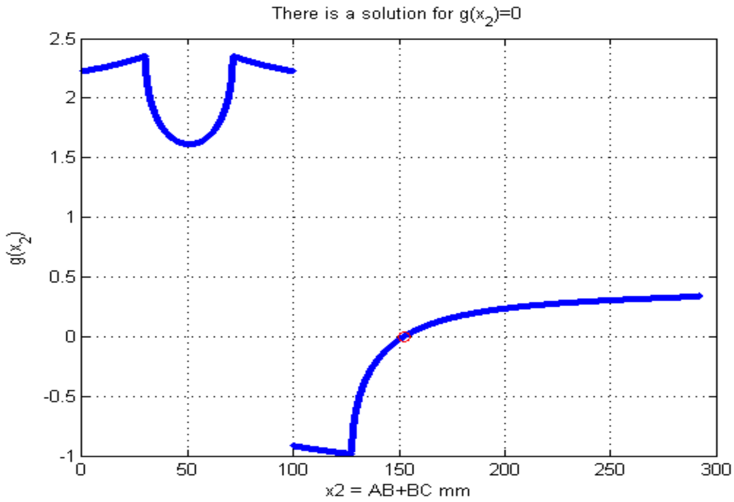


Figure 4. It shows the graphical solution for$\chi_2 \approx 152.2050$.

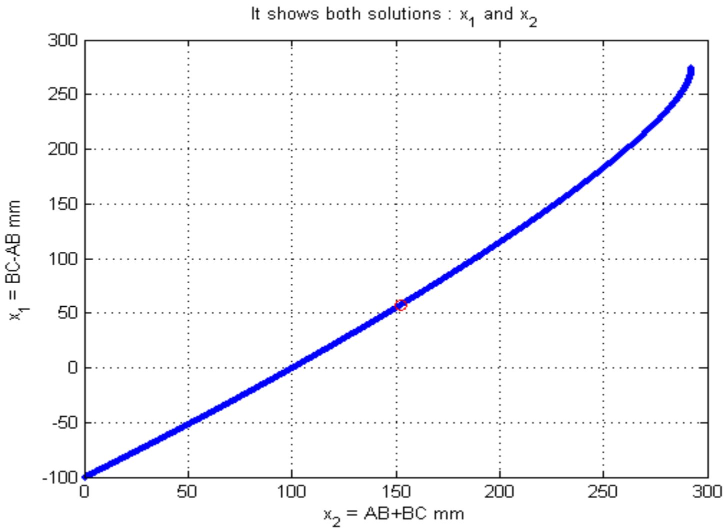


Figure 5. It shows the graphical solution for $\chi_2 \approx 152.2050$ and$\chi_1 \approx 57.6441$.

Found the auxiliary variables $\chi_2 \approx 152.2050$ and$\chi_1 \approx 57.6441$.

$$\chi_1 \equiv BC - AB \approx 57.6441$$
$$\chi_2 \equiv AB + BC \approx 152.2050$$

Then

$$BC \approx 104.92455\, mm \quad , \qquad AB \approx 47.28045\, mm$$

As can see the probe is verified.

Velocity equation:

Given the position equation is terms of Euler angles:

$$ae^{i\theta} + be^{i\alpha} = ce^{i\frac{3}{2}\pi} + de^{i0}$$

Taking time derivate, got:

$$a\, i\frac{d\theta}{dt}e^{i\theta} + bi\frac{d\alpha}{dt}e^{i\alpha} = \frac{d(d)}{dt}$$

$$a\omega_\theta\; i\; e^{i\theta} + b\; i\; \omega_\alpha e^{i\alpha} = V_d$$

$$a\; \omega_\theta\; i\; (\cos\theta + i\sin\theta) + b\; i\; \omega_\alpha(\cos\alpha + i\sin\alpha) = V_d$$

$$a\,\omega_\theta\,i\cos\theta - a\,\omega_\theta\sin\theta + b\,i\,\omega_\alpha\cos\alpha\ - b\,\omega_\alpha\sin\alpha = V_d$$

Split into two simultaneous equations:

$$V_d + b\,\omega_\alpha\sin\alpha = -a\,\omega_\theta\sin\theta$$
$$-\,b\,\omega_\alpha\cos\alpha = \quad a\,\omega_\theta\cos\theta$$

Finally, velocity equation:

$$\begin{bmatrix}1 & b\sin\alpha\\ 0 & -\,b\cos\alpha\end{bmatrix}\begin{bmatrix}V_d\\ \omega_\alpha\end{bmatrix} = \begin{bmatrix}-a\,\omega_\theta\sin\theta\\ a\,\omega_\theta\cos\theta\end{bmatrix}$$

Acceleration equation:

$$a\omega_\theta\,i\,e^{i\theta} + b\ \omega_\alpha i\,e^{i\alpha} = V_d$$

$$\frac{d}{dt}\left(a\omega_\theta\,i\,e^{i\theta}\right) + \frac{d}{dt}\left(b\,\omega_\alpha i\,e^{i\alpha}\right) = \frac{d}{dt}(V_d)$$

$$a\dot{\omega}_\theta\,i\,e^{i\theta} - a{\omega_\theta}^2 e^{i\theta} + b\,\dot{\omega}_\alpha\,i\,e^{i\alpha} - b\,{\omega_\alpha}^2 e^{i\alpha} = \dot{V}_d$$

$$a\,\dot{\omega}_\theta\,i\,(\cos\theta + i\sin\theta) - a\,{\omega_\theta}^2(\cos\theta + i\sin\theta)$$
$$+\,b\,\dot{\omega}_\alpha\,i\,(\cos\alpha + i\sin\alpha) - b\,{\omega_\alpha}^2(\cos\alpha + i\sin\alpha) = \dot{V}_d$$

$$a\dot{\omega}_\theta\,i\cos\theta - a\,\dot{\omega}_\theta\sin\theta - a\,{\omega_\theta}^2\cos\theta - a\,{\omega_\theta}^2\,i\ \sin\theta$$
$$+\,b\,\dot{\omega}_\alpha\,i\cos\alpha -\ b\,\dot{\omega}_\alpha\sin\alpha - b\,{\omega_\alpha}^2\cos\alpha - b\,{\omega_\alpha}^2\,i\ \sin\alpha\ = \dot{V}_d$$

Real part:

$$\dot{V}_d +\ b\,\dot{\omega}_\alpha\sin\alpha =\ -a\,\dot{\omega}_\theta\sin\theta - a\,{\omega_\theta}^2\cos\theta - b\,{\omega_\alpha}^2\cos\alpha$$

Imaginary part:

$$-b\,\dot{\omega}_\alpha\cos\alpha\ \ = a\,\dot{\omega}_\theta\cos\theta - a\,{\omega_\theta}^2\sin\theta - b\,{\omega_\alpha}^2\sin\alpha$$

Finally, acceleration equation in matrix form is given by:

$$\begin{bmatrix}1 & b\sin\alpha\\ 0 & -\,b\cos\alpha\end{bmatrix}\begin{bmatrix}\dot{V}_d\\ \dot{\omega}_\alpha\end{bmatrix} = \begin{bmatrix}-a\dot{\omega}_\theta\sin\theta - a\,{\omega_\theta}^2\cos\theta - b\,{\omega_\alpha}^2\cos\alpha\\ a\dot{\omega}_\theta\cos\theta - a\,{\omega_\theta}^2\sin\theta - b\,{\omega_\alpha}^2\sin\alpha\end{bmatrix}$$

The two unknowns in this matrix equation are the angular acceleration of link BC, $\dot{\omega}_\alpha$ and the linear acceleration of link AC’,$\dot{V}_d$.

Initial conditions:

$h = c\ =\ 30\ mm,a\ =\ 47\ mm,b\ =\ 105\ mm,\theta_{init} = 0$

Local variable	Initial conditions
Crank angle, $\boldsymbol{\theta_{init}}$	$0\ rad$
Connecting rod angle, $\boldsymbol{\alpha_{init}}$	$-16.60154959\ rad$
Speed of Crank,$\boldsymbol{\omega_\theta}$	$376.99111\ rad/s$
Speed of connecting rod, $\boldsymbol{\omega_\alpha}$	$-176.0886892\ rad/s$
Speed of slider-block, $\boldsymbol{V_d}$	$-5282.660677\ mm/s$
Slider-block displacement, $\boldsymbol{d_{init}}$	147.6230589mm

The calculation for above schedule in Appendix B.

Simulink worksheet:

Design a simulation written in Simulink in order to get displacement/time of the offset slider-crank. See figure 6.

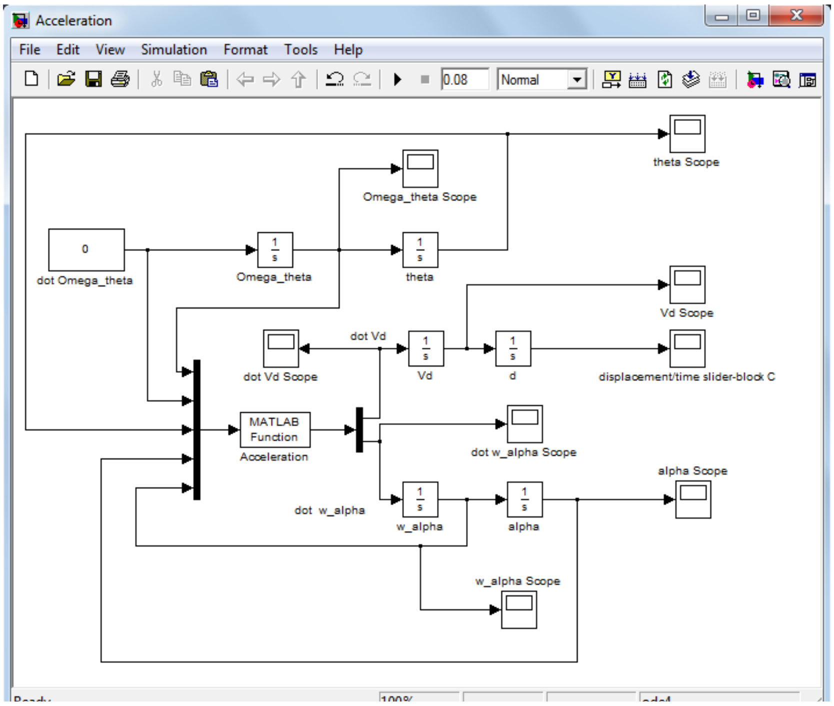


Figure 6: It shows Simulink worksheet of the Acceleration matrix equation.

Result of the acceleration/velocity/position matrix equation for the offset slider-crank:

- Displacement/time offset slider-crank block C.

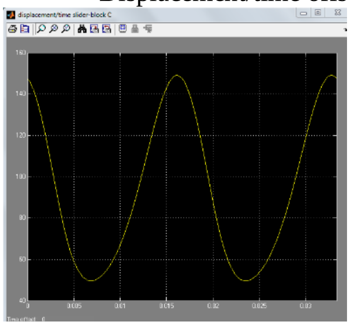


Figure 7: It shows the displacement/time of the slider-block C.

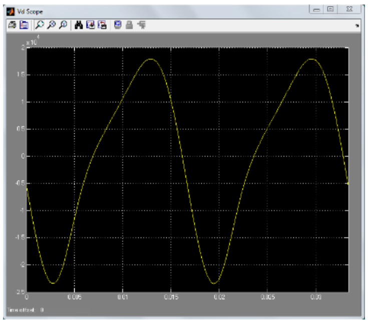


Figure 8: It shows linear velocity of the offset slider-crank at position C, V_d(mm/s) as a function of time (s).

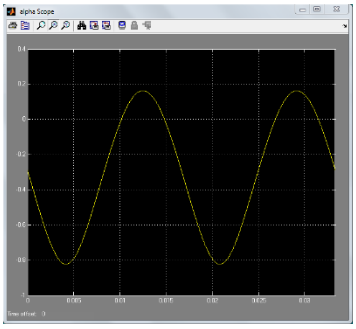
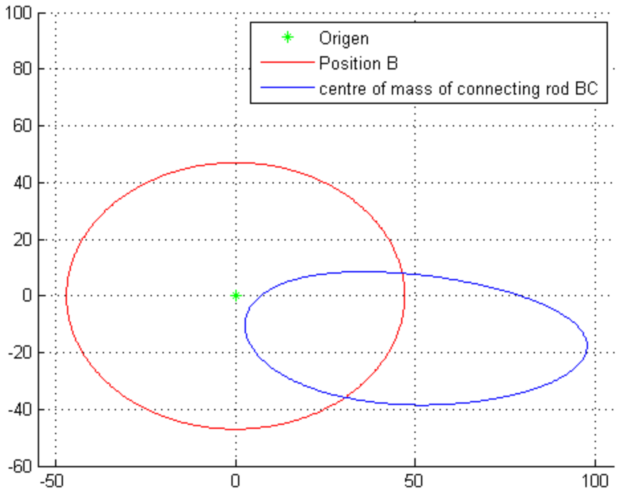


Figure 9: It shows angular acceleration α (rad/s^2) as a function of time(s).

The coupler curve

The coupler curve of the centre of mass of connecting rod BC is calculated from the following polar equation:



It shows the Origen position (green-star), i.e. the position of the point A. The red circle shows the position of point B. The blue curve shows the centre of mass of connecting rod BCi.e. the coupler curve.

Force equation:

In order to determine the forces at the joints and the driving torque needed on the offset crank to provide the specified accelerations. A kinematic analysis must have previously been done and determine all position, velocity and acceleration information for the offset slider crank.

Force equation for link AB, see figure 10:

$$F_{12_x} + F_{32_x} = m_2 a_{G_{2x}}$$
$$F_{12_y} + F_{32_y} = m_2 a_{G_{2y}}$$
$$T_{12} + \left(R_{12_x} F_{12_y} - R_{12_y} F_{12_x}\right) + \left(R_{32_x} F_{32_y} - R_{32_y} F_{32_x}\right) = I_{G_2} \alpha_2$$

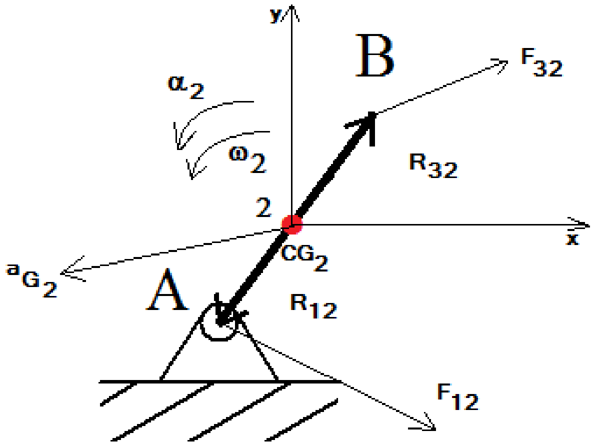


Figure 10: The crank AB has a mass of 0.9 kg and a moment of inertia of 3000 kg.mm^2 about a rotational axis through its centre of mass.

$$m_2 = 0.9\,kg \quad , \quad I_{G_2} = 3000\,kg\,mm^2$$

The vectors $\vec{R}_{32}$ and $\vec{R}_{12}$ are a function of θ angle.

$$\vec{R}_{32} = \tfrac{1}{2} a\, e^{i\,\theta}, \vec{R}_{12} = -\tfrac{1}{2} a\, e^{i\,\theta}$$

$$\vec{a}_{G_2} = \frac{1}{2} a \dot{\omega}_\theta i\, e^{i\,\theta} - \frac{1}{2} a\, \omega_\theta{}^2 e^{i\,\theta}$$

Force equation for link BC, see figure 11:

$$F_{43_x} - F_{32_x} = m_3 a_{G_{3x}}$$

$$F_{43_y} - F_{32_y} = m_3 a_{G_{3y}}$$
$$\left(R_{43_x}F_{43_y} - R_{43_y}F_{43_x}\right) - \left(R_{23_x}F_{32_y} - R_{23_y}F_{32_x}\right) = I_{G_3}\alpha_3$$

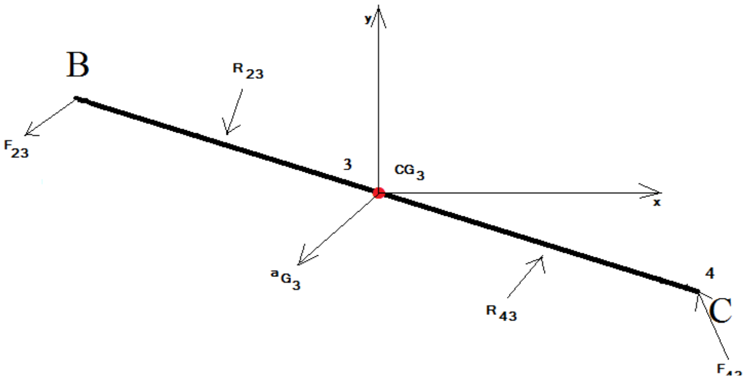


Figure 11: The connecting rod BC has a mass of 0.2 kg and a moment of inertia of 430 kg.mm^2 about a rotational axis through its centre of mass.

$$m_3 = 0.2\,kg,\, I_{G_3} = 430\,kg\,mm^2$$

The vectors $\vec{R}_{23}$ and $\vec{R}_{43}$ is a function of α angle.

$$\vec{R}_{23} = -\frac{1}{2}b\,e^{i\,\alpha} \quad , \; \vec{R}_{43} = \frac{1}{2}b\,e^{i\,\alpha}$$

$$\vec{a}_{G_3} = \frac{1}{2}b\dot{\omega}_\alpha i\, e^{i\,\alpha} - \frac{1}{2}b\,\omega_\alpha{}^2 e^{i\,\alpha}$$

Force equation for block at C, see figure 12:

$$F_{14_x} - F_{43_x} = m_4 a_{G_{4x}}$$
$$F_{14_y} - F_{43_y} = m_4 a_{G_{4y}} = 0$$
$$\left(R_{14_x}F_{14_y} - R_{14_y}F_{14_x}\right) - \left(R_{34_x}F_{43_y} - R_{34_y}F_{43_x}\right) = I_{G_4}\alpha_4$$

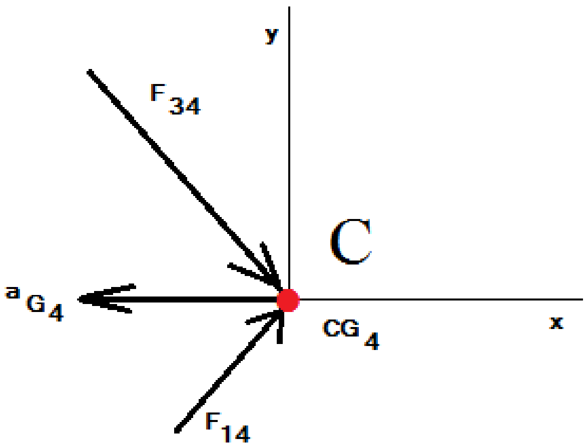


Figure 12: The block C has pure translation movement.

For the offset slider-crank analysed the block is in pure translation against the stationary ground plane; thus it can have no angular acceleration or angular velocity. Also, its linear acceleration has no y component.

$$\alpha_4 = 0\,, \qquad a_{G_{4y}} = 0$$

The slider block C has a mass of 1.2 kg.

$$m_4 = 1.2\ kg$$

The component vector $a_{G_{4_x}}$ is a function of α angle.

$$a_{G_{4_x}} = -\,b\dot{\omega}_\alpha \sin\alpha - b\ {\omega_\alpha}^2 \cos\alpha$$

Additionally,assume friction at the interfaces and weights of all links are negligible $\left(F_{14_x} = 0\right)$ and the mechanism is running under no external load.
Finally, the dynamical system is defined by the following matrix relationship:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -R_{12_y} & R_{12_x} & -R_{32_y} & R_{32_x} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & R_{23_y} & -R_{23_x} & -R_{43_y} & R_{43_x} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ F_{32_x} \\ F_{32_y} \\ F_{43_x} \\ F_{43_y} \\ F_{14_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_{2_x}} \\ m_2 a_{G_{2_y}} \\ I_{G_2}\alpha_2 \\ m_3 a_{G_{3_x}} \\ m_3 a_{G_{3_y}} \\ I_{G_3}\alpha_3 \\ m_4 a_{G_{4_x}} \\ 0 \end{bmatrix}$$

Summery the results: Appendix B
Simulink worksheet to solve force dynamic of the offset slider-crank, see figure 13:

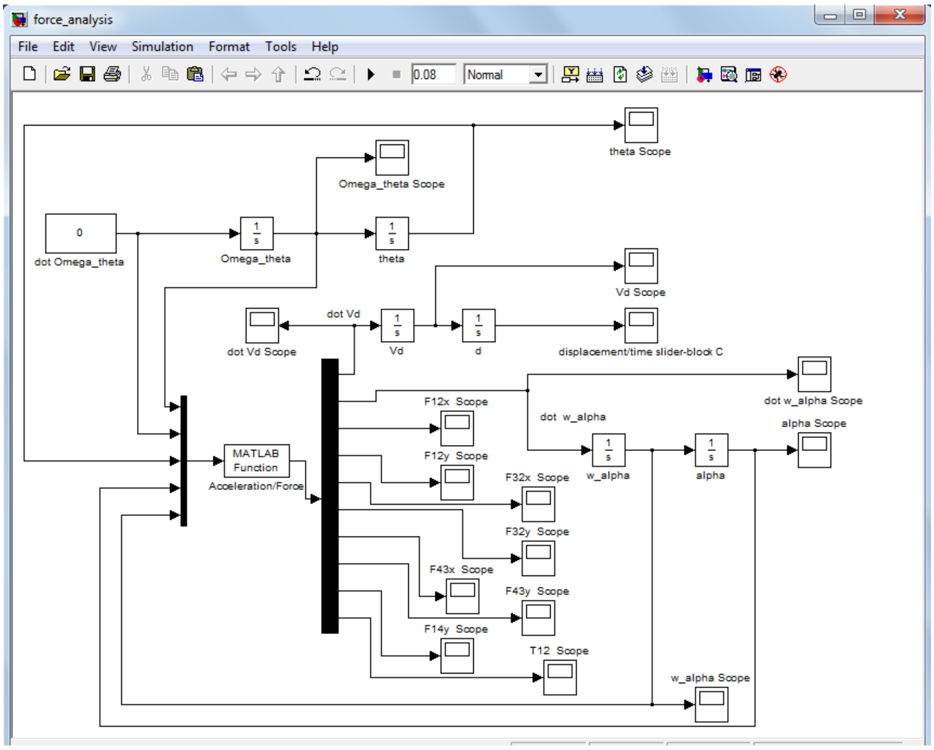


Figure 13: It shows force analysis of the offset slider-crank.

Results of the force analysis:

1. A scope to show the reaction forces/time curve at the pivots A, B and C and the normal reaction force exerted on the slider- block by the guide way.

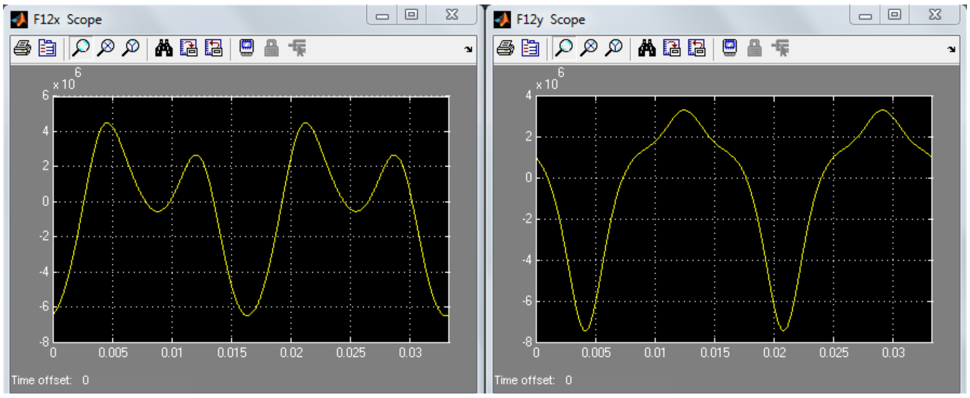


Figure 14: It shows reaction force/time curve at pivot A.

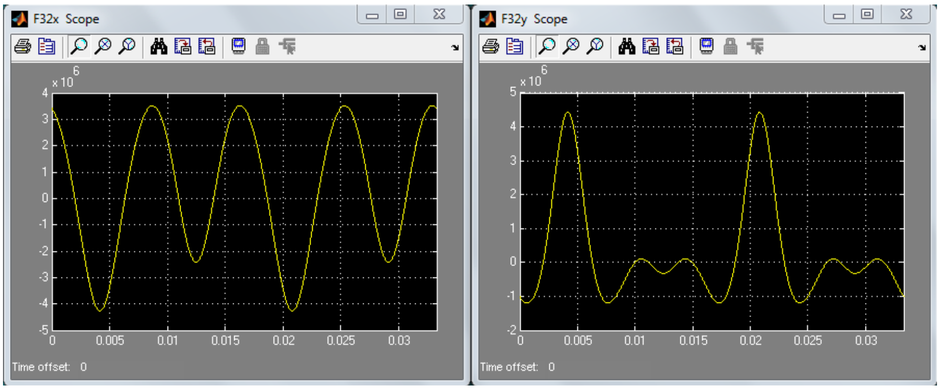


Figure 15: It shows reaction force/time curve at pivot B.

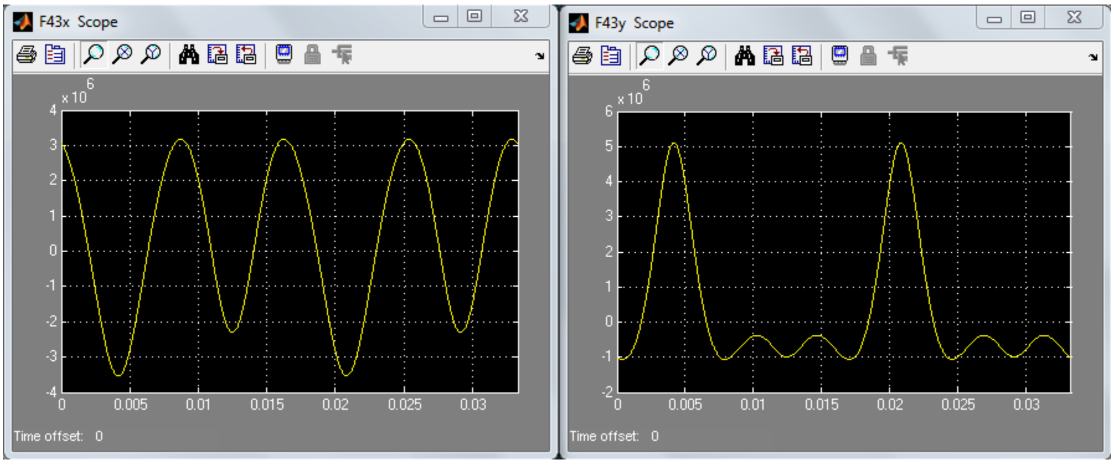


Figure 16: It shows reaction force/time curve at pivot C.

2- A scope to display the displacement/time curve of the slider-block C.	3- A scope to display the torque/time curve of the driving crank AB.

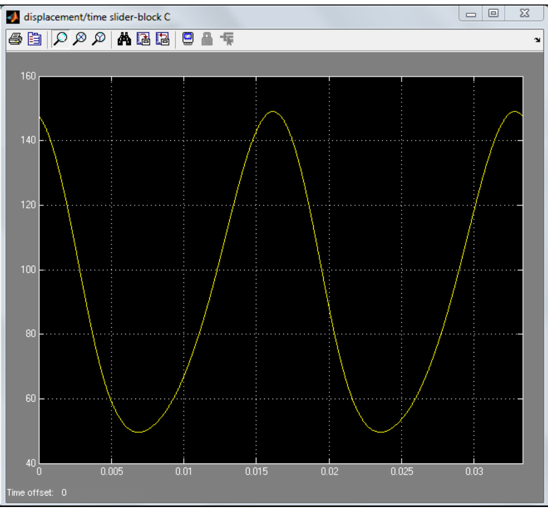
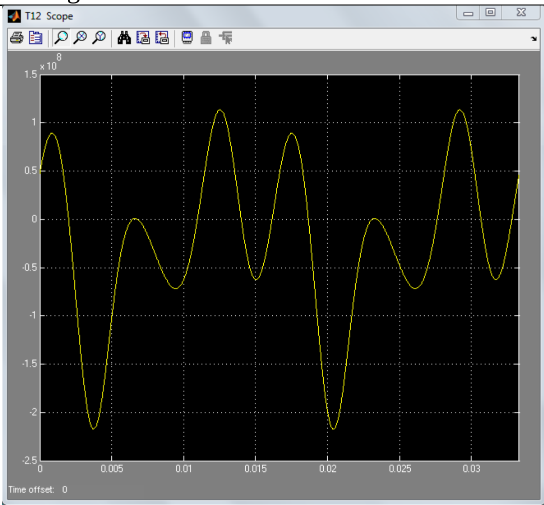

	

Figure 17: It shows the isplacement/time of the slider-block C. Horizontal units are seconds and vertical axis is mm.	Figure 18: It shows torque/time curve of the driving crank AB.

II. CONCLUSION:

The theory of offset slider crank mechanism depends on the different between the forward angle and the return angle of the mechanism (β, α). They graphically display information showing where and when each mechanism is stationary or performing its forward and return strokes. Timing charts allow creators to qualitatively define the required kinematic behaviour of a mechanism [4]. These charts are also used to estimate the velocities and accelerations of certain mechanism. The velocity of a link is the time rate at which its position is changing, while the link's acceleration is the time rate at which its velocity is changing. Both velocity and acceleration are vector quantities, in that they have both magnitude and direction. The analytical method for designing an offset crank slider mechanism is the process to determine generalized relationships among certain lengths, distances, and angles.With these relationships, calculations were done. Simultaneous constraint method is employed. Assembled into a system of linear equations from the kinematic and dynamic analyses to obtain the sparse matrix. Solved by the m-file function results displayed in form of graphs.

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