American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-11, Issue-01, pp-99-103
www.ajer.org
Research Paper Open Access

Resource Distribution Problem

A.N. Voronin, A.S. Savchenko

Voronin Albert Nikolaevich, Doctor tech. Sci., Professor of the National Aviation University, Kiev
Savchenko Alina Stanislavovna, Doctor tech. Sciences, Head of the Department of the National Aviation University, Kiev

***Annotation**. In various subject areas, the problem of such a distribution of the resources of a controlled system between individual elements (objects) is relevant, which ensures the most efficient functioning of the system in given circumstances. The problem of distribution of the given global resource is considered at restrictions from below, applied on partial resources. It is shown, that the problem consists in construction of adequate objective function for optimization of resources distribution process in conditions of their limitation. For the decision of a considered problem the approach of multicriteria optimization with use of the nonlinear trade-off scheme is undertaken. The proposed approach is recommended for a compromise-optimal allocation of resources in a wide range of practical problems. The illustrating example is given.*

***Keywords**: resource allocation, multicriteria optimization, resource constraints, nonlinear trade-off scheme*

Date of Submission: 05-01-2022 Date of acceptance: 17-01-2022

I. INTRODUCTION

In various subject areas, the problem arises of allocating resources between individual objects. We will consider this problem from the standpoint of systems analysis. In the spheres of management and economics, the problem of such a distribution of resources of a controlled system between individual elements (objects) is urgent, which ensures the most efficient functioning of the system in given circumstances. The problem of allocating limited resources is the main problem of the economy. They say that the correct distribution and redistribution of resources is the economy itself. Similar problems arise in other subject areas. The art is to be able to properly allocate limited resources depending on the circumstances.

Often this problem is solved subjectively, based on the experience and professional qualifications of the decision-maker (DM). In simple cases, this approach may be justified. However, with a large number of objects and in critical cases, the cost of an error in a managerial decision rises sharply. It becomes necessary to develop formalized decision support methods for the competent distribution of resources between objects, taking into account all given circumstances.

One such circumstance is usually resources limitation. The most common case is the upper boundedness of the total (global) resource of the system, which is to be distributed between individual objects.

In practical cases, restrictions are imposed not only on the global resource, but also on the partial resources allocated to individual objects. In this case, restrictions can be imposed both from below and from above. The most interesting seems to be the case when partial resources are limited from below. This type of constraint is found in many subject areas. So, doctors and physiologists know that the critical mass of individual organs and tissues, sufficient to support the vital functions of the body, is: liver - 15 percent of the normal volume, kidneys - 25%, erythrocytes - 35%, lungs - 45%, circulating plasma volume - 70%. A decrease in volumes less than the indicated restrictions from below leads to irreversible changes in the body.

Another example of a bottom-up constraint is the distribution of fuel between aircraft when flying to different cities. For each flight there is a lower limit, below which it is pointless to release fuel, the plane simply will not reach its destination. This is the essence of the lower bound for each partial resource. If a given flight receives fuel in excess of a known lower limit, then it has the opportunity to freely maneuver in echelons, bypass a thunderstorm front, go to an alternate airfield, etc. On the other hand, it is also impossible to increase the partial resource indefinitely, there is an upper bound for it. This is understandable if only because each aircraft has a certain tank capacity, more than which it physically cannot take fuel on board.

Such restrictions are either known in advance, or are determined by technical and economic calculations or methods of expert assessments. It is necessary to distinguish between conditional restrictions (when violation of the limits is undesirable) and unconditional restrictions (when their violation is physically impossible). It is easy to see that the sum of the lower limits for all partial resources is the lower limit for the global resource, and the sum of the upper limits restricts the global resource from above.

Taking into account the given set of restrictions, it is required to distribute the global resource of the system among the objects in such a way that the most efficient operation of the entire system as a whole is ensured. The problem lies in the construction of an adequate objective function to optimize the process of resource allocation in conditions of their limitedness. A simple uniform distribution is not suitable in this case, since it can put some objects on the brink of impossibility of their functioning, while other objects will receive an unjustifiably large resource.

In this work, to solve the problem under consideration, a multicriteria optimization approach is taken using a nonlinear trade-off scheme [1-3].

Formulation of the problem

The problem under consideration is relevant for various subject areas. Therefore, we present the problem statement in a general way.

A global resource to be distributed R is given, as well as $n \geq 2$ system elements (objects), to each of which is allocated a partial resource r_i, their totality constitutes a vector $r = \{r_i\}_{i=1}^{n}$. It is clear that

$$\sum_{i=1}^{n} r_i = R. \qquad (1)$$

For each object, the maximum permissible value of the allocated resource $r_{i\min}$ is known (or determined by the method of expert assessments), below which the given object cannot function. Thus, a system of restrictions from below is set

$$r_i \geq r_{i\min}, \sum_{i=1}^{n} r_{i\min} \leq R, i \in [1, n] \qquad (2)$$

On the other hand, for each object, a value $r_{i\max}$ is known that the resource of the object cannot or should not exceed. The upper constraint system has the form

$$r_i \leq r_{i\max}, \sum_{i=1}^{n} r_{i\max} \geq R, i \in [1, n] \qquad (3)$$

It follows from (2) and (3) that

$$r_{i\max} \geq r_i \geq r_{i\min}, i \in [1, n] \qquad (4)$$

and

$$\sum_{i=1}^{n} r_{i\max} \geq R \geq \sum_{i=1}^{n} r_{i\min}. \qquad (5)$$

The formula for the domain of definition of the vector *r* has the form

$$r \in X_r = \{r \mid r_{i\max} \geq r_i \geq r_{i\min}, i \in [1, n]\}. \qquad (6)$$

Let us consider the polar (degenerate) cases of inequality (5). If, $R = \sum_{i=1}^{n} r_{i\,\min}$ then the problem under consideration is reduced to such a distribution of the global resource, in which each object receives its minimum admissible partial resource: $r_i{}^* = r_{i\,\min}, i \in [1, n]$.

If the global resource allows you to fully meet the needs of objects, i.e. $R = \sum_{i=1}^{n} r_{i\,\max}$, then the problem is solved as $r_i{}^* = r_{i\,\max}, i \in [1, n]$.

Thus, in the polar cases of inequality (5), the problem under consideration has trivial solutions. And only if expression (5) becomes a strict inequality

$$\sum_{i=1}^{n} r_{i\,\max} > R > \sum_{i=1}^{n} r_{i\,\min}, \tag{7}$$

the problem of optimizing the allocation of limited resources makes sense.

The optimization problem assumes the presence of an objective function $f(r)$, which extremization gives a solution to the problem under consideration:

$$r^* = \arg \underset{r \in X_r}{extr} f(r). \tag{8}$$

The problem is posed: in conditions (7), determine such partial resources $r^* \in X_r$, at which requirement (1) is fulfilled, and some objective function $f(r)$ acquires an extreme value. The form of $f(r)$ should be selected and justified.

Solution method

The analysis shows that in the problem of optimizing the distribution of limited resources, the upper constraint $r_i \le r_{i\,\max}, i \in [1, n]$ is considered as a simple optimization constraint, the approach to which usually does not threaten anything special for the system. Restriction from below $r_i \ge r_{i\,\min}, i \in [1, n]$ has a completely different meaning. The approach of the resource to this limitation of its own threatens the very possibility of functioning of the corresponding object. We can say that the constraint from below should force the desired objective function to increase the difference between the partial resources and their constraints from below.

Therefore, the expression of the desired objective function must: 1) include explicit constraints from below, 2) penalize the system for approaching partial resources to these constraints, and 3) be differentiable in its arguments. The simplest objective function that satisfies the specified requirements is

$$f(r) = \sum_{i=1}^{n} r_{i\,\min} (r_i - r_{i\,\min})^{-1}. \tag{9}$$

An analysis of formula (9) shows that this is nothing but an expression of the scalar convolution of maximized partial criteria $r_i, i \in [1, n]$ according to a nonlinear trade-off (compromise) scheme (NSC) in the problem of multicriteria optimization [2].

Indeed, in the problem under consideration, resources $r_i, i \in [1, n]$ are of a twofold nature. On the one hand, they can be considered as independent variables, the arguments for the optimization of the objective function $f(r)$. On the other hand, for each of the objects, it is logical to strive to maximize its partial resource, to go as far as possible from a dangerous limitation $r_{i\,\min}$ in order to increase the efficiency of its functioning.

From this point of view, resources $r_i \ge r_{i\,\min}, i \in [1, n]$ can be considered as partial criteria for the quality of functioning of the corresponding objects. These criteria are subject to maximization, they are limited from below, non-negative and contradictory (an increase in one resource is possible only at the expense of a decrease in others).

The NSC concept is based on the “away from constraints” principle. It is assumed that the decision maker's utility function evaluates as preferable those solutions that give the criteria a greater distance from dangerous constraints. The scalar convolution $f(r)$ is a model of the utility function and explicitly includes the difference $r_i - r_{i\min}$ as a characteristic of the intensity of the decision-making situation. This allows criteria to be penalized for approaching their limits.

Based on the foregoing, the problem of vector optimization of the distribution of limited resources, taking into account the isoperimetric constraint for the arguments (1), takes the form

$$r^* = \arg \min_{r \in X_r} f(r) = \arg \min_{r \in X_r} \sum_{i=1}^{n} r_{i\min} (r_i - r_{i\min})^{-1}, \sum_{i=1}^{n} r_i = R . \qquad (10)$$

Problem (10) can be solved both analytically, using the method of indefinite Lagrange multipliers, and by numerical methods, if the analytical solution turns out to be difficult.

The analytical solution provides for the construction of the Lagrange function in the form

$$L(r,\lambda) = f(r) + \lambda (\sum_{i=1}^{n} r_i - R),$$

where λ is the indefinite Lagrange multiplier, and the solution to the system of equations

$$\frac{\partial L(r,\lambda)}{\partial r_i} = 0, i \in [1, n]$$

$$\frac{\partial L(r,\lambda)}{\partial \lambda} = \sum_{i=1}^{n} r_i - R = 0$$

To solve multicriteria problems by numerical methods using the NSC concept and with constraints on arguments and criteria, algorithms have been developed and a computer program has been compiled [2].

Illustrative example

To carry out two long-distance bus routes (n = 2), the bus station has a total volume of fuel R = 12 tons (arbitrary figures). The minimum requirement for the first trip is $r_1 \geq r_{1\min}$ = 2 tons, for the second trip $r_2 \geq r_{2\min}$ = 5 tons. These are lower limits for partial resources. Tank capacity of the first bus $r_{1\max}$ = 7 tons, the second bus $r_{2\max}$ = 10 tons. These are restrictions from above.

Condition (7) in the form of a strict inequality (dimensions are omitted)

$$r_{1\min} + r_{2\min} = 7 < R = 12 < r_{1\max} + r_{2\max} = 17$$

observed. This means that the problem of optimizing the distribution of limited resources can be posed and the solution will be nontrivial.

The task is to obtain an analytical solution for the compromise-optimal distribution of fuel between trips.

We construct the Lagrange function

$$L(r,\lambda) = r_{1\min}(r_1 - r_{1\min})^{-1} + r_{2\min}(r_2 - r_{2\min})^{-1} + \lambda(r_1 + r_2 - R)$$

We get the system of equations

$$\frac{\partial L(r,\lambda)}{\partial r_1} = -r_{1\min}(r_1 - r_{1\min})^{-2} + \lambda = 0$$

$$\frac{\partial L(r,\lambda)}{\partial r_2} = -r_{2\min}(r_2 - r_{2\min})^{-2} + \lambda = 0$$

$$r_1 + r_2 - R = 0$$

Substituting numeric data

$$-2(r_1 - 2)^{-2} + \lambda = 0$$

$$-5(r_2 - 5)^{-2} + \lambda = 0$$

$$r_1 + r_2 - 12 = 0$$

and solving this system by the Gauss method (successive elimination of variables), we obtain

$$r_1{}^* = 3{,}94 \text{ tons}, \; r_2{}^* = 8{,}06 \text{ тons.}$$

The problem is solved under the assumption that the relative importance of both trips for the decision maker is the same. If not, then the weight coefficients are introduced into the objective function, reflecting the individual preferences of the decision maker. These coefficients must be normalized and determined on a simplex:

$$\alpha_1, \alpha_2 \in X_\alpha = \{\alpha_i \,|\, \alpha_i \geq 0, \sum_{i=1}^{n=2} \alpha_i = 1, i \in [1;2]\} .$$

In the case when the global resource is distributed between objects not directly, but through intermediate links, then the system becomes hierarchical [4]. The described approach can be applied in this case as well.

REFERENCES

[1]. Voronin A.N. Vector optimization of hierarchical structures // Problems of Control and Informatics. - 2004. -№ 6. - Pp. 26-34.

[2]. Voronin A.N., Ziatdinov Yu.K., Kozlov A.I. Vector optimization of dynamic systems. - Kiev: Technika, 1999 .-- 284 p.

[3]. Voronin A.N. Nonlinear compromise scheme in multicriteria estimation and optimization problems // Cybernetics and Systems Analysis. - 2009. - No. 4. - Pp. 106-114.

[4]. Prilutsky M.Kh. Multicriteria distribution of a homogeneous resource in hierarchical systems // Automation and Telemechanics. - 1996. - No. 2. - Pp. 24-29.

A.N. Voronin, et. al. "Resource Distribution Problem."*American Journal of Engineering Research (AJER)*, vol. 11(01), 2022, pp. 99-103.